**863.**

**Problem 55.9 (RHK)**

$^{235}\text{U}$ *decays by alpha emission with a half-life of* $7.04\times10^{8}\ \text{y}$. *It also decays* (*rarely*) *by spontaneous fission, and if the alpha decay did not occur, its half-life due to this process alone would be* $3.50\times10^{17}\ \text{y}$. (a) *We have to find the rate with which spontaneous fission decays occur in* $1.00\ \text{g}$ *of* $^{235}\text{U}$. *We have to find the number of alpha-decay events that are there for every spontaneous fission event.*

**Solution:**

The half-life of $^{235}\text{U}$ for alpha emission is $7.04\times10^{8}\ \text{y}$. The disintegration constant for alpha emission will therefore be

$$\lambda_{\text{alpha}} = \frac{\ln 2}{7.04\times10^{8}\times3.156\times10^{7}\ \text{s}} = 3.12\times10^{-17}\ \text{s}^{-1}.$$

The half-life of $^{235}\text{U}$ for spontaneous fission is $3.50\times10^{17}\ \text{y}$.

The disintegration constant for spontaneous fission will therefore be

$$\lambda_{\text{fission}} = \frac{\ln 2}{3.50\times10^{17} \ \times 3.156\times10^{7} \text{ s}} = 6.28\times10^{-26} \text{ s}^{-1}.$$

Number of $^{235}\text{U}$ nuclei in 1.00 g of $^{235}\text{U}$ are

$$N = \frac{6.02\times10^{23}}{235} = 2.56\times10^{21}.$$

The rate of spontaneous fissions in 1.00 g of $^{235}\text{U}$ will therefore be

$$R_{\text{fission}} = N\lambda_{\text{fission}} = 2.56\times10^{21}\times6.28\times10^{-26} \text{ s}^{-1}$$
$$= 16.1\times10^{-5} \text{ s}^{-1}.$$

Therefore, one spontaneous fission event will take place in $1/R_{\text{fission}} = 1/16.1\times10^{-5} \text{ s}^{-1} = 6.21\times10^{3}$ s.

The rate of alpha emission in 1.00 g of $^{235}\text{U}$ will be

$$R_{\text{alpha}} = N\lambda_{\text{alpha}} = 2.56\times10^{21}\times3.12\times10^{-17} \text{ s}^{-1} \text{ s}^{-1}$$
$$= 7.99\times10^{4} \text{ s}^{-1}.$$

The number of alpha decays that will take place in 1.00 g of pure $^{235}\text{U}$ during the interval when one spontaneous fission occurs will therefore be

$$= 7.99\times10^{4} \text{ s}^{-1}\times6.21\times10^{3} \text{ s}$$
$$=4.96\times10^{8}.$$