

860.

Problem 54.87 (RHK)

The nucleus ^{91}Zr ($Z = 40, N = 51$) has a single neutron outside a filled 50-neutrons core. Because 50 is a magic number, this neutron should perhaps be especially loosely bound. We have to calculate (a) its binding energy; (b) the binding energy of the next neutron, which must be extracted from the filled core. (c) We have to find the binding energy per particle for the nucleus. We have to compare these three numbers and discuss. Needed atomic masses are

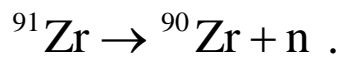
^{91}Zr	90.905644 u	n	1.008665 u
^{90}Zr	89.904703 u	^1H	1.007825 u
^{89}Zr	88.908890 u		

Solution:

(a)

The nucleus ^{91}Zr ($Z = 40, N = 51$) has a single neutron outside a filled 50-neutrons core. We have to calculate its binding energy.

The binding energy of the neutron outside the filled 50-neutrons core can be calculated from the nuclear process

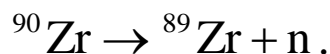


It will be equal to

$$\begin{aligned} & \left(\left(m_{{}^{90}\text{Zr}} - 40m_e + m_n \right) - \left(m_{{}^{91}\text{Zr}} - 40m_e \right) \right) c^2 \\ & = (89.904703 + 1.008665 - 90.905644) \text{ uc}^2 \\ & = 0.007724 \text{ uc}^2 = 0.007724 \times 931.5 \text{ MeV} = 7.19 \text{ MeV}. \end{aligned}$$

(b)

We calculate the energy required to remove the next neutron from the filled shell, that is from ${}^{90}\text{Zr}$. This can be calculated from the nuclear process



It will be equal to

$$\begin{aligned} & \left(\left(m_{{}^{89}\text{Zr}} - 40m_e + m_n \right) - \left(m_{{}^{90}\text{Zr}} - 40m_e \right) \right) c^2 \\ & = (88.908890 + 1.008665 - 89.904703) \text{ uc}^2 \\ & = 0.012852 \text{ uc}^2 = 0.012852 \times 931.5 \text{ MeV} = 11.97 \text{ MeV}. \end{aligned}$$

(c)

We will calculate next the binding energy per nucleon for ${}^{90}\text{Zr}$. It will be equal to

$$\begin{aligned}
& \left(50m_n + 40m_p - (m_{^{90}\text{Zr}} - 40m_e) \right) c^2 / 90 \\
& = (50 \times 1.008665 + 40 \times 1.007825 - 89.908890) \text{ uc}^2 / 90 \\
& = 0.009304 \text{ uc}^2 = 0.009304 \times 931.5 \text{ MeV} = 8.67 \text{ MeV}.
\end{aligned}$$

We note that the binding energy per nucleon of the nucleus ^{90}Zr is 8.67 MeV but the energy required to remove the first neutron from it is 11.97 MeV. As the energy required to remove a neutron from the nuclide ^{91}Zr is 7.19 MeV and the energy required to remove a neutron from the nuclide ^{90}Zr is 11.97 MeV, we note that the neutrons in the filled shell of ^{90}Zr are more tightly bound, because 50 is a nuclear magic number.

