860.

Problem 54.87 (RHK)

The nucleus $^{91}\mathrm{Zr}(Z=40, N=51)$ *has a single neutron outside a filled* 50*-neutrons core. Because* 50 *is a magic number, this neutron should perhaps be especially loosely bound. We have to calculate* (a) *its binding energy*; (b) *the binding energy of the next neutron, which must be extracted from the filled core*. (c) *We have to find the binding energy per particle for the nucleus. We have to compare these three numbers and discuss. Needed atomic masses are*

^{91}Zr	90.905644 u	n	1.008665 u
^{90}Zr	89.904703 u	^{1}H	1.007825 u
^{89}Zr	88.908890 u		

Solution:

(a)

The nucleus $^{91}\mathrm{Zr}(Z=40, N=51)$ has a single neutron outside a filled 50-neutrons core. We have to calculate its binding energy.

The binding energy of the neutron outside the filled 50-neutrons core can be calculated from the nuclear process $^{91}\mathrm{Zr} \rightarrow {}^{90}\mathrm{Zr} + \mathrm{n}$.

It will be equal to

$$\left(\left(m_{^{90}\mathrm{Zr}} - 40m_e + m_{\mathrm{n}}\right) - \left(m_{^{91}\mathrm{Zr}} - 40m_e\right)\right)c^2$$

$$= \left(89.904703 + 1.008665 - 90.905644\right)\ \mathrm{u}c^2$$

$$= 0.007724\ \mathrm{u}c^2 = 0.007724 \times 931.5\ \mathrm{MeV} = 7.19\ \mathrm{MeV}.$$

(b)

We calculate the energy required to remove the next neutron from the filled shell, that is from $^{90}\mathrm{Zr}$. This can be calculated from the nuclear process

$^{90}\mathrm{Zr} \rightarrow {}^{89}\mathrm{Zr} + \mathrm{n}$.

It will be equal to

$$\left(\left(m_{^{89}\mathrm{Zr}} - 40m_e + m_{\mathrm{n}}\right) - \left(m_{^{90}\mathrm{Zr}} - 40m_e\right)\right)c^2$$

$$= \left(88.908890 + 1.008665 - 89.904703\right)\ \mathrm{u}c^2$$

$$= 0.012852\ \mathrm{u}c^2 = 0.012852 \times 931.5\ \mathrm{MeV} = 11.97\ \mathrm{MeV}.$$

(c)

We will calculate next the binding energy per nucleon for $^{90}\mathrm{Zr}$. It will be equal to

$$\left(50m_{\mathrm{n}} + 40m_{\mathrm{p}} - \left(m_{^{90}\mathrm{Zr}} - 40m_e\right)\right)c^2 \Big/ 90$$
$$= \left(50 \times 1.008665 + 40 \times 1.007825 - 89.908890\right)\ \mathrm{u}c^2 \Big/ 90$$
$$= 0.009304\ \mathrm{u}c^2 = 0.009304 \times 931.5\ \mathrm{MeV} = 8.67\ \mathrm{MeV}.$$

We note that the binding energy per nucleon of the nucleus $^{90}\mathrm{Zr}$ is 8.67 MeV but the energy required to remove the first neutron from it is 11.97 MeV. As the energy required to remove a neutron from the nuclide $^{91}\mathrm{Zr}$ is 7.19 MeV and the energy required to remove a neutron from the nuclide $^{90}\mathrm{Zr}$ is 11.97 MeV, we note that the neutrons in the filled shell of $^{90}\mathrm{Zr}$ are more tightly bound, because 50 is a nuclear magic number.