859.

<u>Problem 54.86 (RHK)</u>

The nuclide $^{208}\mathrm{Pb}$ *is "doubly magic" in that both its proton number* $Z(=82)$ *and its neutron number* $N(=126)$ *represent filled nucleon shells. An additional proton would yield* $^{209}\mathrm{Bi}$ *and an additional neutron would yield* $^{209}\mathrm{Pb}$. *These "extra" nucleons should be easier to remove than a proton or a neutron from the filled shells of* $^{208}\mathrm{Pb}$.

(a) *We have to calculate the energy required to move the "extra" proton from* $^{209}\mathrm{Bi}$ *and compare it with the energy required to remove a proton from the filled proton shell of* $^{208}\mathrm{Pb}$.

(b) *We have to calculate the energy required to remove the "extra" neutron from* $^{209}\mathrm{Pb}$ *and compare it with the energy required to remove a neutron from the filled neutron shell of* $^{208}\mathrm{Pb}$. *We have to answer whether our results agree with expectation. Needed atomic mass data is given in the following table*:

Nuclide	*Z*	*N*	*Atomic Mass* (u)
^{209}Bi	82 + 1	126	208.980374
^{208}Pb	82	126	207.976627
^{207}Tl	82 − 1	126	206.977404
^{209}Pb	82	126 + 1	208.981065
^{207}Pb	82	126 − 1	206.975872

The masses of the proton and the neutron are 1.007276 u *and* 1.008665 u, *respectively.*

<u>Solution:</u>

(a)

We have to calculate the energy required to move the "extra" proton from $^{209}\mathrm{Bi}$ and compare it with the energy required to remove a proton from the filled proton shell of $^{208}\mathrm{Pb}$.

The energy required to move the "extra" proton from $^{209}\mathrm{Bi}$ can be calculated by considering the nuclear process

$^{209}\mathrm{Bi} \rightarrow {}^{208}\mathrm{Pb} + \mathrm{p}$.

It will be equal to

$$\left(\left(m_{^{208}\mathrm{Pb}} - 82m_e + m_\mathrm{P}\right) - \left(m_{^{209}\mathrm{Bi}} - 83m_e\right)\right)c^2$$
$$= \left(207.976627 + 1.007276 + 0.51/931.5 - 208.980374\right)\mathrm{u}c^2$$
$$= 0.004080\ \mathrm{u}c^2 = 0.004080 \times 931.5\ \text{MeV=3.80 MeV.}$$

We calculate next the energy required to remove a proton from the filled proton shell of $^{208}\mathrm{Pb}$.

It will be given by the nuclear process

$$^{208}\mathrm{Pb} \rightarrow {}^{207}\mathrm{Tl} + \mathrm{p}\ .$$

It will be equal to

$$\left(\left(m_{^{207}\mathrm{Tl}} - 81m_e + m_\mathrm{P}\right) - \left(m_{^{208}\mathrm{Pb}} - 82m_e\right)\right)c^2$$
$$= \left(206.977404 + 1.007276 + 0.51/931.5 - 207.976627\right)\mathrm{u}c^2$$
$$= 0.008107\ \mathrm{u}c^2 = 0.008107 \times 931.5\ \text{MeV=7.55 MeV.}$$

(b)

We have to calculate the energy required to remove the "extra" neutron from $^{209}\mathrm{Pb}$ and compare it with the energy required to remove a neutron from the filled neutron shell of $^{208}\mathrm{Pb}$.

The energy required to remove the "extra" neutron from $^{209}\mathrm{Pb}$ can be calculated from the nuclear process

$$^{209}\mathrm{Pb} \rightarrow {}^{208}\mathrm{Pb+n}\,.$$

It will be equal to

$$\left(\left(m_{^{208}\mathrm{Pb}} - 82m_e + m_\mathrm{n}\right) - \left(m_{^{209}\mathrm{Pb}} - 82m_e\right)\right)c^2$$
$$= \left(207.976627 + 1.008665 - 208.981065\right)\mathrm{u}c^2$$
$$= 0.00423\ \mathrm{u}c^2 = 0.00423 \times 931.5\ \mathrm{MeV} = 3.94\ \mathrm{MeV}.$$

The energy required to remove a neutron from the filled neutron shell of $^{208}\mathrm{Pb}$ can be calculated from the nuclear process

$$^{208}\mathrm{Pb} \rightarrow {}^{207}\mathrm{Pb} + \mathrm{n}.$$

It will be equal to

$$\left(\left(m_{^{207}\mathrm{Pb}} - 82m_e + m_\mathrm{n}\right) - \left(m_{^{208}\mathrm{Pb}} - 82m_e\right)\right)c^2$$
$$= \left(206.975872 + 1.008665 - 207.976627\right)\mathrm{u}c^2$$
$$= 0.00791\ \mathrm{u}c^2 = 0.00791 \times 931.5\ \mathrm{MeV} = 7.37\ \mathrm{MeV}.$$

We note that our results agree with expectation that because the nuclide $^{208}\mathrm{Pb}$ is “doubly magic” in that both its proton number $Z\left(=82\right)$ and its neutron number $N\left(=126\right)$ represent filled nucleon shells, protons are more tightly bound in it than in the nuclide $^{209}\mathrm{Bi}$, and that neutrons are more tightly bound in it than in the nuclide $^{209}\mathrm{Pb}$.