

859.

Problem 54.86 (RHK)

The nuclide ^{208}Pb is “doubly magic” in that both its proton number $Z(=82)$ and its neutron number $N(=126)$ represent filled nucleon shells. An additional proton would yield ^{209}Bi and an additional neutron would yield ^{209}Pb . These “extra” nucleons should be easier to remove than a proton or a neutron from the filled shells of ^{208}Pb .

(a) We have to calculate the energy required to move the “extra” proton from ^{209}Bi and compare it with the energy required to remove a proton from the filled proton shell of ^{208}Pb .

(b) We have to calculate the energy required to remove the “extra” neutron from ^{209}Pb and compare it with the energy required to remove a neutron from the filled neutron shell of ^{208}Pb . We have to answer whether our results agree with expectation. Needed atomic mass data is given in the following table:

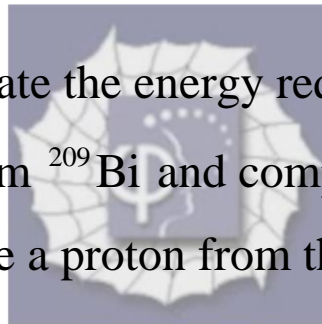
<i>Nuclide</i>	<i>Z</i>	<i>N</i>	<i>Atomic Mass (u)</i>
^{209}Bi	$82 + 1$	126	208.980374
^{208}Pb	82	126	207.976627
^{207}Tl	$82 - 1$	126	206.977404
^{209}Pb	82	$126 + 1$	208.981065
^{207}Pb	82	$126 - 1$	206.975872

The masses of the proton and the neutron are 1.007276 u and 1.008665 u, respectively.

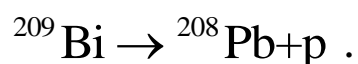
Solution:

(a)

We have to calculate the energy required to move the “extra” proton from ^{209}Bi and compare it with the energy required to remove a proton from the filled proton shell of ^{208}Pb .



The energy required to move the “extra” proton from ^{209}Bi can be calculated by considering the nuclear process



It will be equal to

$$\begin{aligned}
& \left((m_{^{208}\text{Pb}} - 82m_e + m_p) - (m_{^{209}\text{Bi}} - 83m_e) \right) c^2 \\
& = (207.976627 + 1.007276 + 0.51/931.5 - 208.980374) \text{uc}^2 \\
& = 0.004080 \text{uc}^2 = 0.004080 \times 931.5 \text{ MeV} = 3.80 \text{ MeV}.
\end{aligned}$$

We calculate next the energy required to remove a proton from the filled proton shell of ^{208}Pb .

It will be given by the nuclear process



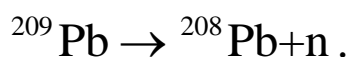
It will be equal to

$$\begin{aligned}
& \left((m_{^{207}\text{Tl}} - 81m_e + m_p) - (m_{^{208}\text{Pb}} - 82m_e) \right) c^2 \\
& = (206.977404 + 1.007276 + 0.51/931.5 - 207.976627) \text{uc}^2 \\
& = 0.008107 \text{uc}^2 = 0.008107 \times 931.5 \text{ MeV} = 7.55 \text{ MeV}.
\end{aligned}$$

(b)

We have to calculate the energy required to remove the “extra” neutron from ^{209}Pb and compare it with the energy required to remove a neutron from the filled neutron shell of ^{208}Pb .

The energy required to remove the “extra” neutron from ^{209}Pb can be calculated from the nuclear process



It will be equal to

$$\begin{aligned}
& \left((m_{^{208}\text{Pb}} - 82m_e + m_n) - (m_{^{209}\text{Pb}} - 82m_e) \right) c^2 \\
&= (207.976627 + 1.008665 - 208.981065) \text{uc}^2 \\
&= 0.00423 \text{uc}^2 = 0.00423 \times 931.5 \text{ MeV} = 3.94 \text{ MeV}.
\end{aligned}$$

The energy required to remove a neutron from the filled neutron shell of ^{208}Pb can be calculated from the nuclear process



It will be equal to

$$\begin{aligned}
& \left((m_{^{207}\text{Pb}} - 82m_e + m_n) - (m_{^{208}\text{Pb}} - 82m_e) \right) c^2 \\
&= (206.975872 + 1.008665 - 207.976627) \text{uc}^2 \\
&= 0.00791 \text{uc}^2 = 0.00791 \times 931.5 \text{ MeV} = 7.37 \text{ MeV}.
\end{aligned}$$

We note that our results agree with expectation that because the nuclide ^{208}Pb is “doubly magic” in that both its proton number $Z (= 82)$ and its neutron number $N (= 126)$ represent filled nucleon shells, protons are more tightly bound in it than in the nuclide ^{209}Bi , and that neutrons are more tightly bound in it than in the nuclide ^{209}Pb .

