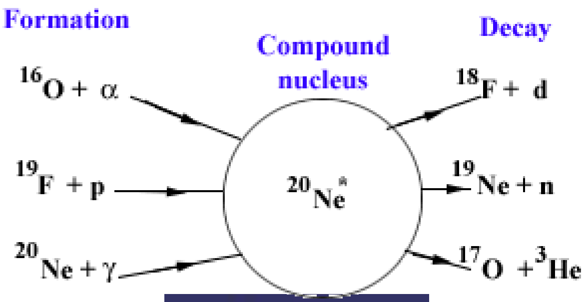
858.

Problem 54.84 (RHK)

We consider the three formation modes, as shown in the figure, for the compound nucleus $^{20}\mathrm{Ne}^*$.



We have to find the energies that (a) α *particle,* (b) *the proton, and* (c) *the gamma-ray photon must have to provide* 25.00 MeV *of excitation energy to the compound nucleus. Needed atomic masses are*

$^{20}\mathrm{Ne}$	19.992435 u	α	4.002603 u
$^{19}\mathrm{F}$	18.998403 u	$^{1}\mathrm{H}$	1.007825 u
$^{16}\mathrm{O}$	15.994915 u		

Solution:

The atomic number of Ne is 10, so its atom contains 10 electrons.

As the compound nucleus ${}^{20}\mathrm{Ne}^{*}$ has 25 MeV of excitation energy and in the centre-of-mass frame of the nuclear reaction the compound nucleus ${}^{20}\mathrm{Ne}^{*}$ will be at rest, therefore the needed total energy must be

$$E_{\mathrm{cm}} = \left(19.992435\times 931.5\text{-}10\times 0.51\right)\ \mathrm{MeV} + 25\ \mathrm{MeV}$$
$$= 20.013798\ \mathrm{u}c^2.$$

For answering this problem, we will use relativistic mechanics. In the collision of two particles of masses m_1 and m_2 the energy of the particle 1 in the rest frame of particle 2 is given by

$$E_{1\ \mathrm{lab}} = \frac{E_{\mathrm{cm}}{}^2 - \left(m_1^2 c^4\right) - \left(m_2^2 c^4\right)}{2 m_2 c^2}.$$

(a)

α particle

We consider the nuclear reaction

$$\alpha + {}^{16}\mathrm{O} \to {}^{20}\mathrm{Ne}^{*}.$$

The rest mass energy of an α particle will be

$$m_\alpha c^2 = \left(4.002603 - 2\times 0.51/931.5\right)\ \mathrm{u}c^2$$
$$= 4.001508\ \mathrm{u}c^2.$$

The rest mass energy of an ${}^{16}\mathrm{O}$ nucleus will be

$$m_{^{16}\mathrm{O}}c^2 = (15.994915 - 8\times 0.51/931.5)\ \mathrm{u}c^2$$
$$= 15.990535\ \mathrm{u}c^2.$$

Therefore,

$$E_{\alpha\ \mathrm{lab}} = \frac{E_{\mathrm{cm}}{}^2 - (m_\alpha^2 c^4) - (m_{^{16}\mathrm{O}}^2 c^4)}{2m_{^{16}\mathrm{O}}c^2}$$
$$= \frac{(20.013798)^2 - (4.001508)^2 - (15.990535)^2}{2\times(15.990535)}\mathrm{u}c^2$$
$$= 4.028722\ \mathrm{u}c^2.$$

The kinetic energy of the α particle in the lab frame will be

$$KE_\alpha = (4.028722 - 4.001508)\ \mathrm{u}c^2 = 0.027214\times 931.5\ \mathrm{MeV}$$
$$= 25.349\ \mathrm{MeV}.$$

(b)

Proton

We consider the nuclear reaction

$$p + {}^{19}\mathrm{F} \to {}^{20}\mathrm{Ne}^*.$$

The rest mass energy of a proton is

$$m_\mathrm{p}c^2 = 1.007276\ \mathrm{u}c^2.$$

The atomic number of F is 9.

The rest mass energy of a ${}^{19}\mathrm{F}$ nucleus will be

$$m_{^{19}\mathrm{F}}c^2 = \left(18.998403 - 9\times 0.51/931.5\right)\ \mathrm{u}c^2$$
$$= 18.993475\ \mathrm{u}c^2.$$

Therefore,

$$E_{\mathrm{p\ lab}} = \frac{E_{\mathrm{cm}}{}^2 - \left(m_{\mathrm{p}}^2 c^4\right) - \left(m_{^{19}\mathrm{F}}^2 c^4\right)}{2m_{^{19}\mathrm{F}}c^2}$$
$$= \frac{\left(20.013798\right)^2 - \left(1.007276\right)^2 - \left(18.993475\right)^2}{2\times\left(18.993475\right)}\mathrm{u}c^2$$
$$= 1.021019\ \mathrm{u}c^2.$$

The kinetic energy of the proton in the laboratory frame will be

$$KE_{\mathrm{P}} = \left(1.021019 - 1.007276\right)\ \mathrm{u}c^2 = 0.013733\ \mathrm{u}c^2$$
$$= 12.80\ \mathrm{MeV}.$$

(c)

Photon

We consider the nuclear reaction

$$\gamma + {}^{20}\mathrm{Ne} \to {}^{20}\mathrm{Ne}^*.$$

The rest mass energy of a photon is zero.

The rest mass energy of a ${}^{20}\mathrm{Ne}$ nucleus is

$$m_{^{20}\mathrm{Ne}} = \left(19.992435 - 10\times 0.51/931.5\right)\ \mathrm{u}c^2$$
$$= 19.986959\ \mathrm{u}c^2.$$

Therefore,

$$E_{\gamma\ \mathrm{lab}} = \frac{{E_{\mathrm{cm}}}^{2} - \left(m_{\gamma}^{2} c^{4}\right) - \left(m_{{}^{20}\mathrm{Ne}}^{2} c^{4}\right)}{2 m_{{}^{20}\mathrm{Ne}} c^{2}}$$

$$= \frac{\left(20.013798\right)^{2} - \left(19.986959\right)^{2}}{2 \times \left(19.986959\right)} \mathrm{u} c^{2}$$

$$= 0.026857\ \mathrm{u} c^{2} = 0.026857 \times 931.5\ \mathrm{meV} = 25.017\ \mathrm{MeV}.$$