855.

Problem 54.78 (RHK)

We consider the reaction $X(a,b)Y$, *in which* X *is taken to be at rest in the laboratory reference frame. In an endothermic reaction* $(Q<0)$, *the interacting particles* a *and* X *must have a kinetic energy, measured in the centre-of-mass frame, of at least* $|Q|$ *if the reaction is to "go". We have to show using the result*

$$K_{\text{cm}} = K_{\text{lab}}\left(\frac{m_X}{m_X + m_a}\right)$$

that the threshold energy for the particle a, *measured in the laboratory frame, is*

$$K_{\text{th}} = |Q|\frac{m_X + m_a}{m_X}.$$

We have to answer whether it is reasonable that K_{th} *should be greater than* $K_{\text{cm}} = |Q|$.

Solution:

We note that

$$|Q| = (m_b + m_Y - m_a - m_X)c^2.$$

In the centre-of-mass frame the sum of the momentum of the particles a and X is zero. So for the endothermic reaction to "go" through the total energy in the centre-of-mass frame has to be such that

$$KE_{\text{cm}} + \left(m_a + m_X\right)c^2 = \left(m_b + m_Y\right)c^2.$$

Or

$$KE_{\text{cm}} = \left|Q\right|.$$

The required minimum kinetic energy of the incident particle a will therefore be

$$K_{\text{lab}} = K_{\text{cm}} \times \left(\frac{m_X + m_a}{m_X} \right)$$

and for $K_{\text{cm}} = \left|Q\right|$, we note that

$$K_{\text{th}} = \left|Q\right| \left(\frac{m_X + m_a}{m_X} \right).$$

It is reasonable that K_{th} should be greater than $K_{\text{cm}} = \left|Q\right|$, because in the centre-mass-frame in addition to particles a and X their centre-of-mass is also moving with respect to the laboratory frame and carries kinetic energy.