

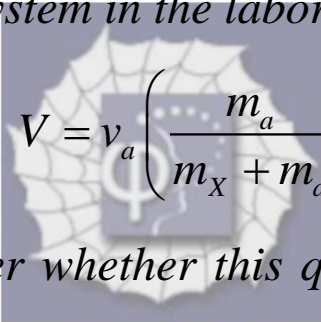
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Problem 54.77 (RHK)

We consider the reaction $X(a,b)Y$, in which X is taken to be at rest in the laboratory reference frame. The initial kinetic energy in this frame is

$$K_{\text{lab}} = \frac{1}{2} m_a v_a^2.$$

We have to show (a) that the initial velocity of the centre of mass is of the system in the laboratory frame is


$$V = v_a \left(\frac{m_a}{m_X + m_a} \right).$$

We have to answer whether this quantity is changed by the reaction. (b) We have to show that the initial kinetic energy, viewed now in a reference attached to the centre of mass of the two particles, is given by

$$K_{\text{cm}} = K_{\text{lab}} \left(\frac{m_X}{m_X + m_a} \right).$$

We have to answer whether this quantity is changed by the reaction. (c) In the reaction $^{90}\text{Zr}(d,p)^{91}\text{Zr}$ the kinetic energy of the deuteron, measured in the laboratory

frame, is 15.9 MeV. We have to find $v_a (= v_d)$, V , and K_{cm} . We may ignore the small relativistic effects.

Solution:

(a)

The centre of mass is defined by the equation

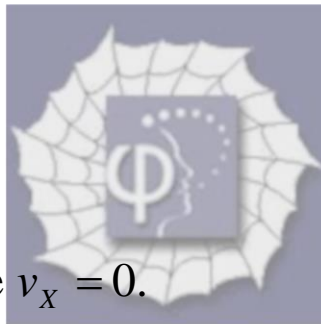
$$X_{\text{cm}} = \frac{m_a x_a + m_X x_X}{m_a + m_X}.$$

Therefore, the velocity of the centre of mass dX_{cm}/dt

will be

$$V = \frac{m_a v_a}{m_a + m_X},$$

as in the lab frame $v_X = 0$.



The velocity of the centre of mass is a property of the frame of reference and does not change by the reaction.

(b)

In the centre of mass frame, the velocity of the particle a will be

$$\begin{aligned} (v_a)_{\text{cm}} &= v_a - V = v_a - \frac{m_a v_a}{m_a + m_X} \\ &= \frac{m_X v_a}{m_a + m_X}, \end{aligned}$$

and that of the particle X will be

$$(v_X)_{\text{cm}} = -V = -v_a \left(\frac{m_a}{m_X + m_a} \right).$$

Therefore, the kinetic energy in the centre of mass frame will be

$$\begin{aligned} K_{\text{cm}} &= \frac{1}{2} m_a (v_a)_{\text{cm}}^2 + \frac{1}{2} m_X (v_X)_{\text{cm}}^2 \\ &= \frac{1}{2} m_a \left(\frac{m_X v_a}{m_a + m_X} \right)^2 + \frac{1}{2} m_X \left(\frac{m_a v_a}{m_a + m_X} \right)^2 \\ &= \frac{1}{2} m_a m_X v_a^2 \frac{(m_a + m_X)}{(m_a + m_X)^2} \\ &= \frac{1}{2} m_a v_a^2 \left(\frac{m_X}{m_X + m_a} \right) = K_{\text{lab}} \left(\frac{m_X}{m_X + m_a} \right). \end{aligned}$$

If $Q \neq 0$, the K_{cm} will be changed by the reaction.

(c)

In the reaction $^{90}\text{Zr}(d,p)^{91}\text{Zr}$ the kinetic energy of the deuteron, measured in the laboratory frame, is 15.9 MeV.

We have to find $v_a (= v_d)$, V , and K_{cm} .

The velocity of the deuteron in the lab frame will be

$$\begin{aligned}
 v_d &= \sqrt{2E_d/m_d} \\
 &= \left(\frac{2 \times 15.9 \times 1.6 \times 10^{-13}}{2.014102 \times 1.6605 \times 10^{-27}} \right) \text{ms}^{-1} \\
 &= 3.90 \times 10^7 \text{ms}^{-1}.
 \end{aligned}$$

And, the velocity of the centre of mass will be

$$\begin{aligned}
 V &= v_d \left(\frac{m_d}{m_{90\text{Zr}} + m_a} \right) = \frac{2}{90+2} v_d = 0.0217 \times 3.9 \times 10^7 \text{ms}^{-1} \\
 &= 8.5 \times 10^5 \text{ms}^{-1}.
 \end{aligned}$$

The kinetic energy in the centre of mass frame will be

$$\begin{aligned}
 K_{\text{cm}} &= K_{\text{lab}} \left(\frac{m_{90\text{Zr}}}{m_{90\text{Zr}} + m_d} \right) = 15.9 \times \frac{90}{92} \text{MeV} \\
 &= 15.55 \text{MeV}.
 \end{aligned}$$

