854.

Problem 54.77 (RHK)

We consider the reaction $X(a,b)Y$, *in which* X *is taken to be at rest in the laboratory reference frame. The initial kinetic energy in this frame is*

$$K_{\mathrm{lab}} = \tfrac{1}{2} m_a v_a^2.$$

We have to show (a) *that the initial velocity of the centre of mass is of the system in the laboratory frame is*

$$V = v_a \left(\frac{m_a}{m_X + m_a} \right).$$

We have to answer whether this quantity is changed by the reaction. (b) *We have to show that the initial kinetic energy, viewed now in a reference attached to the centre of mass of the two particles, is given by*

$$K_{\mathrm{cm}} = K_{\mathrm{lab}} \left(\frac{m_X}{m_X + m_a} \right).$$

We have to answer whether this quantity is changed by the reaction. (c) *In the reaction* $^{90}\mathrm{Zr}(\mathrm{d,p})\,^{91}\mathrm{Zr}$ *the kinetic energy of the deuteron, measured in the laboratory* *frame, is* 15.9 MeV. *We have to find* $v_a\left(=v_{\mathrm{d}}\right)$, V, *and* K_{cm}. *We may ignore the small relativistic effects.*

Solution:

(a)

The centre of mass is defined by the equation

$$X_{\mathrm{cm}}=\frac{m_a x_a+m_X x_X}{m_a+m_X}.$$

Therefore, the velocity of the centre of mass dX_{cm}/dt will be

$$V=\frac{m_a v_a}{m_a+m_X},$$

as in the lab frame $v_X=0$.

The velocity of the centre of mass is a property of the frame of reference and does not change by the reaction.

(b)

In the centre of mass frame, the velocity of the particle *a* will be

$$\left(v_a\right)_{\mathrm{cm}}=v_a-V=v_a-\frac{m_a v_a}{m_a+m_X}$$

$$=\frac{m_X v_a}{m_a+m_X},$$

and that of the particle X will be

$$(v_X)_{\text{cm}} = -V = -v_a\left(\frac{m_a}{m_X + m_a}\right).$$

Therefore, the kinetic energy in the centre of mass frame will be

$$\begin{aligned}
K_{\text{cm}} &= \tfrac{1}{2}m_a (v_a)_{\text{cm}}^2 + \tfrac{1}{2}m_X (v_X)_{\text{cm}}^2 \\
&= \tfrac{1}{2}m_a\left(\frac{m_X v_a}{m_a + m_X}\right)^2 + \tfrac{1}{2}m_X\left(\frac{m_a v_a}{m_a + m_X}\right)^2 \\
&= \tfrac{1}{2}m_a m_X v_a{}^2 \frac{(m_a + m_X)}{(m_a + m_X)^2} \\
&= \tfrac{1}{2}m_a v_a{}^2\left(\frac{m_X}{m_X + m_a}\right) = K_{\text{lab}}\left(\frac{m_X}{m_X + m_a}\right).
\end{aligned}$$

If $Q \neq 0$, the K_{cm} will be changed by the reaction.

(c)

In the reaction $^{90}\text{Zr}(\text{d,p})^{91}\text{Zr}$ the kinetic energy of the deuteron, measured in the laboratory frame, is 15.9 MeV. We have to find $v_a\,(= v_{\text{d}})$, V, and K_{cm}.

The velocity of the deuteron in the lab frame will be

$$v_{\mathrm{d}} = \sqrt{2E_{\mathrm{d}}/m_{\mathrm{d}}}$$

$$= \left(\frac{2\times 15.9\times 1.6\times 10^{-13}}{2.014102\times 1.6605\times 10^{-27}} \right) \text{ ms}^{-1}$$

$$= 3.90\times 10^{7} \text{ ms}^{-1}.$$

And, the velocity of the centre of mass will be

$$V = v_d \left(\frac{m_d}{m_{^{90}\mathrm{Zr}} + m_a} \right) = \frac{2}{90+2} v_d = 0.0217\times 3.9\times 10^{7} \text{ ms}^{-1}$$

$$= 8.5\times 10^{5} \text{ ms}^{-1}.$$

The kinetic energy in the centre of mass frame will be

$$K_{\mathrm{cm}} = K_{\mathrm{lab}} \left(\frac{m_{^{90}\mathrm{Zr}}}{m_{^{90}\mathrm{Zr}} + m_{\mathrm{d}}} \right) = 15.9\times \frac{90}{92} \text{ MeV}$$

$$= 15.55 \text{ MeV}.$$