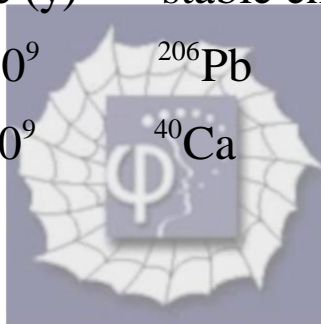


852.

**Problem 54.69 (RHK)**

*A rock, recovered from far underground, is found to contain 860  $\mu\text{g}$  of  $^{238}\text{U}$ , 150  $\mu\text{g}$  of  $^{206}\text{Pb}$ , and 1.60 mg of  $^{40}\text{Ca}$ . We have to calculate the amount of  $^{40}\text{K}$  it is likely to contain. The needed half-lives are as follows:*

| parent           | half-life (y)      | stable end point  |
|------------------|--------------------|-------------------|
| $^{238}\text{U}$ | $4.47 \times 10^9$ | $^{206}\text{Pb}$ |
| $^{40}\text{K}$  | $1.28 \times 10^9$ | $^{40}\text{Ca}$  |



**Solution:**

We assume that the amount of  $^{206}\text{Pb}$  and  $^{40}\text{Ca}$  present in the rock are due to the radio active decays of  $^{238}\text{U}$ , and  $^{40}\text{K}$ , respectively, and were trapped inside the rock in  $t$  y because of the decay of their mother nuclides. We estimate the number of nuclides in 860  $\mu\text{g}$  of  $^{238}\text{U}$ , 150  $\mu\text{g}$  of  $^{206}\text{Pb}$ , and 1.60 mg of  $^{40}\text{Ca}$ .

$$N_{^{238}\text{U}} = \frac{6.02 \times 10^{23} \times 860 \times 10^{-6} \text{ g}}{238 \text{ g}} = 2.17 \times 10^{18},$$

$$N_{^{206}\text{Pb}} = \frac{6.02 \times 10^{23} \times 150 \times 10^{-6} \text{ g}}{206 \text{ g}} = 4.38 \times 10^{17},$$

and

$$N_{^{40}\text{Ca}} = \frac{6.02 \times 10^{23} \times 1.60 \times 10^{-3} \text{ g}}{40 \text{ g}} = 2.41 \times 10^{19}.$$

We use the radioactive decay law

$$N(t) = N(0)e^{-\lambda t},$$

in the following for calculating the age of the rock and the number of nuclides of  $^{40}\text{K}$  that the rock contained when it was formed.

The disintegration constant of  $^{238}\text{U}$  decay is

$$\lambda_{^{238}\text{U}} = \frac{\ln 2}{4.47 \times 10^9 \text{ y}} = 1.55 \times 10^{-10} \text{ y}^{-1}.$$

The disintegration constant of  $^{40}\text{K}$  decay is

$$\lambda_{^{40}\text{K}} = \frac{\ln 2}{1.28 \times 10^9 \text{ y}} = 5.42 \times 10^{-10} \text{ y}^{-1}.$$

The nuclides of  $^{238}\text{U}$  that were there in the rock when it was formed will be the sum of the nuclides of  $^{238}\text{U}$  and those of  $^{206}\text{Pb}$  present now. We thus note that

$$\begin{aligned} N_{^{238}\text{U}}(0) &= N_{^{238}\text{U}}(t) + N_{^{206}\text{Pb}} = 2.17 \times 10^{18} + 4.38 \times 10^{17} \\ &= 26.08 \times 10^{17}. \end{aligned}$$

Therefore, the age of the rock can be found from the relation

$$\lambda_{238\text{U}} t = 1.55 \times 10^{-10} \times t = \ln \left( \frac{N_{238\text{U}}(0)}{N_{238\text{U}}(t)} \right)$$

$$= \ln \left( \frac{26.08 \times 10^{17}}{21.7 \times 10^{17}} \right),$$

or

$$1.55 \times 10^{-10} \times t = 0.183,$$

or

$$t = \frac{0.183}{1.55} \times 10^{10} = 1.18 \times 10^9.$$

The age of the rock is  $1.18 \times 10^9$  y.

Let the number of nuclides of  $^{40}\text{K}$  that the rock is likely to contain be  $m$  mg. The number of nuclides of  $^{40}\text{K}$  in  $m$  mg will be

$$N_{40\text{K}}(t) = \frac{6.02 \times 10^{23} \times m \times 10^{-3} \text{ g}}{40 \text{ g}} = 1.51 \times 10^{19} m .$$

We use again the fact that the sum of nuclides of  $^{40}\text{K}$  and those of  $^{40}\text{Ca}$  present now will be the number of nuclides of  $^{40}\text{K}$  present at the time of the formation of the rock, which is  $1.18 \times 10^9$  y ago. We have

$$e^{\lambda_{40\text{K}} t} = e^{5.42 \times 10^{-10} \times 1.18 \times 10^9} = \frac{N_{40\text{K}}(0) + N_{40\text{Ca}}(0)}{N_{40\text{K}}(0)}$$

or

$$e^{0.639} = \frac{1.51m + 2.41}{1.51m},$$

or

$$1.89 = 1 + \frac{1.60}{m},$$

or

$$m = \frac{1.60}{0.89} = 1.79.$$

We find that the amount of  $^{40}\text{K}$  present in the rock is 1.79 mg.

