852.

Problem 54.69 (RHK)

A rock, recovered from far underground, is found to contain 860 μg *of* $^{238}\mathrm{U}$, 150 μg *of* $^{206}\mathrm{Pb}$, *and* 1.60 mg *of* $^{40}\mathrm{Ca}$. *We have to calculate the amount of* $^{40}\mathrm{K}$ *it is likely to contain. The needed half-lives are as follows*:

parent	half-life (y)	stable end point
$^{238}\mathrm{U}$	4.47×10^{9}	$^{206}\mathrm{Pb}$
$^{40}\mathrm{K}$	1.28×10^{9}	$^{40}\mathrm{Ca}$

Solution:

We assume that the amount of $^{206}\mathrm{Pb}$ and $^{40}\mathrm{Ca}$ present in the rock are due to the radio active decays of $^{238}\mathrm{U}$, and $^{40}\mathrm{K}$, respectively, and were trapped inside the rock in t y because of the decay of their mother nuclides. We estimate the number of nuclides in 860 μg of $^{238}\mathrm{U}$, 150 μg of $^{206}\mathrm{Pb}$, and 1.60 mg of $^{40}\mathrm{Ca}$.

$$N_{^{238}\mathrm{U}} = \frac{6.02\times10^{23}\times860\times10^{-6}\ \mathrm{g}}{238\ \mathrm{g}} = 2.17\times10^{18},$$

$$N_{^{206}\mathrm{Pb}} = \frac{6.02\times10^{23}\times150\times10^{-6}\ \mathrm{g}}{206\ \mathrm{g}} = 4.38\times10^{17},$$

and

$$N_{^{40}\mathrm{Ca}} = \frac{6.02\times10^{23}\times1.60\times10^{-3}\ \mathrm{g}}{40\ \mathrm{g}} = 2.41\times10^{19}\ .$$

We use the radioactive decay law

$$N(t) = N(0)e^{-\lambda t},$$

in the following for calculating the age of the rock and the number of nuclides of $^{40}\mathrm{K}$ that the rock contained when it was formed.

The disintegration constant of $^{238}\mathrm{U}$ decay is

$$\lambda_{^{238}\mathrm{U}} = \frac{\ln 2}{4.47\times10^{9}\ \mathrm{y}} = 1.55\times10^{-10}\ \mathrm{y}^{-1}.$$

The disintegration constant of $^{40}\mathrm{K}$ decay is

$$\lambda_{^{40}\mathrm{K}} = \frac{\ln 2}{1.28\times10^{9}\ \mathrm{y}} = 5.42\times10^{-10}\ \mathrm{y}^{-1}.$$

The nuclides of $^{238}\mathrm{U}$ that were there in the rock when it was formed will be the sum of the nuclides of $^{238}\mathrm{U}$ and those of $^{206}\mathrm{Pb}$ present now. We thus note that

$$\begin{aligned} N_{^{238}\mathrm{U}}(0) = N_{^{238}\mathrm{U}}(t) + N_{^{206}\mathrm{Pb}} &= 2.17\times10^{18} + 4.38\times10^{17} \\ &= 26.08\times10^{17}. \end{aligned}$$

Therefore, the age of the rock can be found from the relation

$$\lambda_{^{238}\mathrm{U}}t = 1.55\times10^{-10}\times t = \ln\left(\frac{N_{^{238}\mathrm{U}}(0)}{N_{^{238}\mathrm{U}}(t)}\right)$$

$$= \ln\left(\frac{26.08\times10^{17}}{21.7\times10^{17}}\right),$$

or

$$1.55\times10^{-10}\times t = 0.183,$$

or

$$t = \frac{0.183}{1.55}\times10^{10} = 1.18\times10^{9}.$$

The age of the rock is 1.18×10^{9} y.

Let the number of nuclides of $^{40}\mathrm{K}$ that the rock is likely to contain be *m* mg. The number of nuclides of $^{40}\mathrm{K}$ in *m* mg will be

$$N_{^{40}\mathrm{K}}(t) = \frac{6.02\times10^{23}\times m\times10^{-3}\ \mathrm{g}}{40\ \mathrm{g}} = 1.51\times10^{19}m\ .$$

We use again the fact that the sum of nuclides of $^{40}\mathrm{K}$ and those of $^{40}\mathrm{Ca}$ present now will be the number of nuclides of $^{40}\mathrm{K}$ present at the time of the formation of the rock, which is 1.18×10^{9} y ago. We have

$$e^{\lambda_{^{40}\mathrm{K}}t} = e^{5.42\times10^{-10}\times1.18\times10^{9}} = \frac{N_{^{40}\mathrm{K}}(0) + N_{^{40}\mathrm{Ca}}(0)}{N_{^{40}\mathrm{K}}(0)}$$

or

$$e^{0.639} = \frac{1.51m + 2.41}{1.51m},$$

or

$$1.89=1+\frac{1.60}{m},$$

or

$$m = \frac{1.60}{0.89} = 1.79.$$

We find that the amount of $^{40}\mathrm{K}$ present in the rock is 1.79 mg.