851.

Problem 54.68 (RHK)

A particular rock is thought to be 260 million years *old. If it contains* 3.71 mg *of* $^{238}\mathrm{U}$, *we have to find how much* $^{206}\mathrm{Pb}$ *should it contain.*

Solution:

A particular rock is thought to be 260 million years old. It contains 3.71 mg of $^{238}\mathrm{U}$. Therefore, the number of $^{238}\mathrm{U}$ nuclides contained in the rock sample will be

$$N_{^{238}\mathrm{U}} = \frac{6.02\times10^{23}\times3.71\times10^{-3}\ \mathrm{g}}{238\ \mathrm{g}}$$

$$= 9.384\times10^{18}.$$

The half-life of $^{238}\mathrm{U}$ for radioactive decay to stable endpoint $^{206}\mathrm{Pb}$ is 4.47×10^{9} y. The disintegration constant will be

$$\lambda_{^{238}\mathrm{U}} = \frac{\ln 2}{4.47\times10^{9}\ \times3.156\times10^{7}\ \mathrm{s}} = 4.91\times10^{-18}\ \mathrm{s}^{-1}.$$

The number of $^{238}\mathrm{U}$ nuclides in the rock sample 260 million years ago will be

$$N_{^{238}\mathrm{U}} \exp(\lambda t)$$
$$= 9.384\times10^{18} \times \exp\left(4.91\times10^{-18} \times 260\times10^{6} \times 3.156\times10^{7}\right)$$
$$= 9.384\times10^{18} \times \exp(0.0403) = 9.769\times10^{18}.$$

The number of $^{206}\mathrm{Pb}$ nuclides in the rock sample will therefore be

$$N_{^{238}\mathrm{U}} \exp(\lambda t) - N_{^{238}\mathrm{U}} = 9.769\times10^{18} - 9.384\times10^{18}$$
$$= 0.385\times10^{18}.$$

The amount in g of the $^{206}\mathrm{Pb}$ in the rock sample will be

$$m_{^{206}\mathrm{Pb}} = \frac{206\times0.385\times10^{18}}{6.02\times10^{23}}\ \mathrm{g} = 0.13\ \mathrm{mg}.$$