850.

<u>Problem 54.67 (RHK)</u>

Two radioactive materials that are unstable to alpha decay, $^{238}\mathrm{U}$ *and* $^{232}\mathrm{Th}$*, and one that is unstable to beta decay,* $^{40}\mathrm{K}$*, are sufficiently abundant in granite to contribute significantly to the heating of the Earth through the decay energy produced. The alpha-unstable isotopes give rise to decay chains that stop at stable lead isotopes.* $^{40}\mathrm{K}$ *has a single beta decay. The decay information is as follows*:

Parent Nuclide	Decay Mode	Half-life (y)	Stable Endpoint	Q (MeV)	f (ppm)
$^{238}\mathrm{U}$	α	4.47×10^{9}	$^{206}\mathrm{Pb}$	51.7	4
$^{232}\mathrm{Th}$	α	1.41×10^{10}	$^{208}\mathrm{Pb}$	42.7	13
$^{40}\mathrm{K}$	β	1.28×10^{9}	$^{40}\mathrm{Ca}$	1.32	4

Q *is the total energy released in the decay of one parent nucleus to the final stable endpoint and* f *is the abundance of the isotope in kilograms per kilogram of granite*; ppm *means parts per million. We have to show* (a) *that these materials give rise to a total heat production of* 988 pW *for each kilogram of granite.* (b) *Assuming that there is* 2.7×10^{22} kg *of granite in a*

20-km *thick, spherical shell around the Earth, we have to estimate the power this will produce over the whole Earth. We will compare this with the total solar power intercepted by the Earth,* 1.7×10^{7} W.

Solution:

(a)

^{238}U:

As the abundance of ^{238}U in 1 kg of granite is 4×10^{-6} kg, the number of ^{238}U nuclides in 1 kg of granite will be

$$N_{^{238}\mathrm{U}} = \frac{6.02\times10^{23}\times4\times10^{-3}}{238\ \mathrm{g}} = 1.01\times10^{19}.$$

The half-life of ^{238}U for radioactive decay to stable endpoint ^{206}Pb is 4.47×10^{9} y. The disintegration constant will be

$$\lambda_{^{238}\mathrm{U}} = \frac{\ln 2}{4.47\times10^{9}\ \times3.156\times10^{7}\ \mathrm{s}} = 4.91\times10^{-18}\ \mathrm{s}^{-1}.$$

The decay rate of ^{238}U nuclides in 1 kg of granite will be

$$R_{^{238}\mathrm{U}} = \lambda_{^{238}\mathrm{U}}N_{^{238}\mathrm{U}} = 1.01\times10^{19}\times4.91\times10^{-18}\ \mathrm{s}^{-1}$$
$$= 49.6\ \mathrm{s}^{-1}.$$

For this decay Q is 51.7 MeV. Therefore, energy released per second due to radioactive decay of ^{238}U nuclides contained in 1 kg of granite will be

$$P_{^{238}\text{U}} = 51.7 \times 1.6 \times 10^{-13} \times 49.6 \text{ J s}^{-1} \text{ per kg of granite}$$
$$= 410.3 \text{ pW per kg of granite.}$$

^{232}Th:

We consider next the energy production in 1 kilogram of granite because of radioactive decay of ^{232}Th nuclides contained in it.

As the abundance of ^{232}Th in 1 kg of granite is 13×10^{-6} kg, the number of ^{232}Th nuclides in 1 kg of granite will be

$$N_{^{232}\text{Th}} = \frac{6.02 \times 10^{23} \times 13 \times 10^{-3}}{232 \text{ g}} = 3.37 \times 10^{19}.$$

The half-life of ^{232}Th for radioactive decay to stable endpoint ^{206}Pb is 1.41×10^{10} y. Its disintegration constant will be

$$\lambda_{^{232}\text{Th}} = \frac{\ln 2}{1.41 \times 10^{10} \times 3.156 \times 10^{7} \text{ s}} = 1.56 \times 10^{-18} \text{ s}^{-1}.$$

The decay rate of ^{232}Th nuclides in 1 kg of granite will be

$$R_{^{232}\text{Th}} = \lambda_{^{232}\text{Th}} N_{^{232}\text{Th}} = 3.37 \times 10^{19} \times 1.56 \times 10^{-18} \text{ s}^{-1}$$
$$= 52.6 \text{ s}^{-1}.$$

For this decay Q is 42.7 MeV. Therefore, energy released per second due to radioactive decay of ^{232}Th nuclides contained in 1 kg of granite will be

$$P_{^{232}\text{Th}} = 42.7 \times 1.6 \times 10^{-13} \times 52.6 \text{ J s}^{-1} \text{ per kg of granite}$$
$$= 359.4 \text{ pW per kg of granite.}$$

^{40}K:

We consider next the energy production in 1 kilogram of granite because of radioactive decay of ^{40}K nuclides contained in it.

As the abundance of ^{40}K nuclides in 1 kg of granite is 4×10^{-6} kg, the number of ^{40}K nuclides in 1 kg of granite will be

$$N_{^{40}\text{K}} = \frac{6.02 \times 10^{23} \times 4 \times 10^{-3}}{40 \text{ g}} = 6.02 \times 10^{19}.$$

The half-life of ^{40}K for beta decay is 1.28×10^{9} y. The disintegration constant will be

$$\lambda_{^{40}\text{K}} = \frac{\ln 2}{1.28 \times 10^{9} \times 3.156 \times 10^{7} \text{ s}} = 1.72 \times 10^{-17} \text{ s}^{-1}.$$

The decay rate of ^{40}K nuclides in 1 kg of granite will be

$$R_{^{40}\mathrm{K}} = \lambda_{^{40}\mathrm{K}} N_{^{40}\mathrm{K}} = 1.72\times10^{-17} \times 6.02\times10^{19} \text{ s}^{-1}$$

$$= 1035 \text{ s}^{-1}.$$

For this decay Q is 1.32 MeV. Therefore, energy released per second due to radioactive decay of $^{40}\mathrm{K}$ nuclides contained in 1 kg of granite will be

$$P_{^{40}\mathrm{K}} = 1.32\times1.6\times10^{-13}\times1035 \text{ J s}^{-1} \text{ per kg of granite}$$

$$= 218.6 \text{ pW per kg of granite.}$$

We thus find that the total energy released per second in 1 kilogram of granite due to radioactive disintegrations will be

$$P = P_{^{238}\mathrm{U}} + P_{^{232}\mathrm{Th}} + P_{^{40}\mathrm{K}}$$

$$= (410.3 + 359.4 + 218.6) \text{ pW per kg of granite}$$

$$= 988.3 \text{ pW per kg of granite.}$$

(b)

Assuming that there is 2.7×10^{22} kg of granite in a 20-km thick, spherical shell around the Earth, we find that the power produced over the whole Earth because of radioactive decays will be

$$P_{\text{Earth - radioactivity}} = 988 \times 10^{-12} \times 2.7 \times 10^{22} \text{ W}$$

$$= 2.67 \times 10^{13} \text{ W}.$$

The total solar power intercepted by the Earth is 1.7×10^{7} W, which is $0.64 \times 10^{-6} P_{\text{Earth - radioactivity}}$.