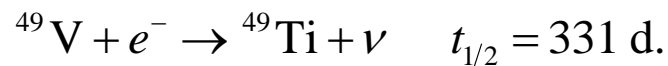


842.

Problem 54.52 (RHK)

Some radionuclides decay by capturing one of their own atomic electrons, a K-electron, say. An example is



(a) *We have to show that the disintegration energy Q for this process is given by*

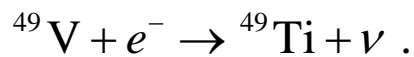
$$Q = (m_{\text{V}} - m_{\text{Ti}})c^2 - E_{\text{K}},$$

where m_{V} and m_{Ti} are the atomic masses of ${}^{49}\text{V}$ and ${}^{49}\text{Ti}$, respectively, and E_{K} is the binding energy of the vanadium K-electron.

(b) *We have to find the disintegration energy Q for the decay of ${}^{49}\text{V}$ by K-electron capture. The needed data are $m_{\text{V}} = 48.948517 \text{ u}$, $m_{\text{Ti}} = 48.94781 \text{ u}$, and $E_{\text{K}} = 5.47 \text{ keV}$.*

Solution:

We have to find the disintegration energy Q for the decay of ${}^{49}\text{V}$ by K-electron capture. The beta decay process is



We denote by m_{V} and m_{Ti} the atomic masses of ${}^{49}\text{V}$ and ${}^{49}\text{Ti}$, respectively, and let E_K be the binding energy of the vanadium K -electron. We denote by m_{V}' the mass of the ${}^{49}\text{V}$ nucleus, and by m_{Ti}' the mass of the ${}^{49}\text{Ti}$ nucleus.

The Q value for the beta decay of ${}^{49}\text{V}$ nucleus by electron capture will therefore be

$$\begin{aligned} Q &= \left(m_{\text{V}}' + m_e - m_{\text{Ti}}' \right) c^2 \\ &= \left(m_{\text{V}}' + m_e + 22m_e \right) c^2 - \left(m_{\text{Ti}}' + 22m_e \right) c^2 \\ &= \left(m_{\text{V}} c^2 - E_K \right) - m_{\text{Ti}} c^2 . \end{aligned}$$

We have used the result that m_{V} atom has 23 electrons and m_{Ti} atom has 22 electrons. We have ignored the differences in the binding energies of the atomic electrons in m_{V} atom and those in m_{Ti} atom, except by taking into account that the extra electron in m_{V} atom has less energy by an amount E_K , as the energy in the K -shell is $-E_K$.

(b)

We use the data for calculating the Q for the beta decay of ^{49}V by K-electron capture. We have

$$\begin{aligned} Q &= (m_{\text{V}} - m_{\text{Ti}})c^2 - E_{\text{K}} \\ &= (48.948517 \text{ u} - 48.94781 \text{ u})c^2 - 5.47 \text{ keV} \\ &= 0.000707 \times 931.5 \text{ MeV} - 5.47 \text{ keV} \\ &= 658.57 \text{ keV} - 5.47 \text{ keV} = 653.1 \text{ keV}. \end{aligned}$$

