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Problem 54.52 (RHK)

Some radionuclides decay by capturing one of their own atomic electrons, a K-electron, say. An example is

$$^{49}\mathrm{V} + e^{-} \rightarrow {}^{49}\mathrm{Ti} + \nu \qquad t_{1/2} = 331\ \mathrm{d}.$$

(a) *We have to show that the disintegration energy* Q *for this process is given by*

$$Q = \left(m_{\mathrm{V}} - m_{\mathrm{Ti}}\right)c^2 - E_K,$$

where m_{V} *and* m_{Ti} *are the atomic masses of* $^{49}\mathrm{V}$ and $^{49}\mathrm{Ti}$, *respectively, and* E_K *is the binding energy of the vanadium K-electron.*

(b) *We have to find the disintegration energy* Q *for the decay of* $^{49}\mathrm{V}$ *by K-electron capture. The needed data are* $m_{\mathrm{V}} = 48.948517\ \mathrm{u}$, $m_{\mathrm{Ti}} = 48.94781\ \mathrm{u}$, *and* $E_K = 5.47\ \mathrm{keV}$.

Solution:

We have to find the disintegration energy Q for the decay of $^{49}\mathrm{V}$ by *K*-electron capture. The beta decay process is

$^{49}\mathrm{V} + e^- \rightarrow {}^{49}\mathrm{Ti} + \nu$.

We denote by m_V and m_Ti the atomic masses of $^{49}\mathrm{V}$ and $^{49}\mathrm{Ti}$, respectively, and let E_K be the binding energy of the vanadium *K*-electron. We denote by m_V' the mass of the $^{49}\mathrm{V}$ nucleus, and by m_Ti' the mass of the $^{49}\mathrm{Ti}$ nucleus.

The Q value for the beta decay of $^{49}\mathrm{V}$ nucleus by electron capture will therefore be

$$Q = \left(m_\mathrm{V}' + m_e - m_\mathrm{Ti}'\right)c^2$$

$$= \left(m_\mathrm{V}' + m_e + 22m_e\right)c^2 - \left(m_\mathrm{Ti}' + 22m_e\right)c^2$$

$$= \left(m_\mathrm{V}c^2 - E_K\right) - m_\mathrm{Ti}c^2 .$$

We have used the result that m_V atom has 23 electrons and m_Ti atom has 22 electrons. We have ignored the differences in the binding energies of the atomic electrons in m_V atom and those in m_Ti atom, except by taking into account that the extra electron in m_V atom has less energy by an amount E_K, as the energy in the *K*-shell is $-E_K$.

(b)

We use the data for calculating the Q for the beta decay of $^{49}\mathrm{V}$ by K-electron capture. We have

$$Q = \left(m_{\mathrm{V}} - m_{\mathrm{Ti}}\right)c^2 - E_K$$
$$= \left(48.948517\ \mathrm{u} - 48.94781\ \mathrm{u}\right)c^2 - 5.47\ \mathrm{keV}$$
$$= 0.000707 \times 931.5\ \mathrm{MeV} - 5.47\ \mathrm{keV}$$
$$= 658.57\ \mathrm{keV} - 5.47\ \mathrm{keV} = 653.1\ \mathrm{keV}.$$