841.

Problem 54.51 (RHK)

The radionuclide $^{11}\mathrm{C}$ *decays according to*

$$^{11}\mathrm{C} \rightarrow {}^{11}\mathrm{B} + e^{+} + \nu, \quad t_{1/2} = 20.3 \text{ min.}$$

The maximum energy of the positron spectrum is 960.8 keV. *We have to show* (a) *that the disintegration energy* Q *for this process is given by*

$$Q = \left(m_{\mathrm{C}} - m_{B} - 2m_{e}\right)c^{2},$$

where m_{C} *and* m_{B} *are atomic masses of* $^{11}\mathrm{C}$ *and* $^{11}\mathrm{B}$, *respectively and* m_{e} *is the electron* (*positron*) *mass*. (b) *given that* $m_{\mathrm{C}} = 11.011433 \text{ u}$, $m_{\mathrm{B}} = 11.009305 \text{ u}$, *and* $m_{e} = 0.0005486 \text{ u}$, *we have to calculate* Q *and compare it with the maximum energy of the positron spectrum, given above*.

Solution:

The radionuclide $^{11}\mathrm{C}$ decays according to

$$^{11}\mathrm{C} \rightarrow {}^{11}\mathrm{B} + e^{+} + \nu, \quad t_{1/2} = 20.3 \text{ min.}$$

The atomic mass of $^{11}\mathrm{C}$, $m_{\mathrm{C}} = 11.011433\ \mathrm{u}$, the atomic mass of $^{11}\mathrm{B}$, $m_{\mathrm{B}} = 11.009305\ \mathrm{u}$, and the electron (positron) mass, $m_e = 0.0005486\ \mathrm{u}$. We may recall that $^{11}\mathrm{C}$ atom has six electrons and that the $^{11}\mathrm{B}$ atom has 5 electrons. In calculating the Q value for the decay process $^{11}\mathrm{C} \rightarrow {}^{11}\mathrm{B} + e^{+} + \nu$ we have to note that it involves the mass of nuclei of these atoms and not their atomic mass. Therefore, Q value for the decay process $^{11}\mathrm{C} \rightarrow {}^{11}\mathrm{B} + e^{+} + \nu$ is given by the equation

$$Q = \left(\left(m_{\mathrm{C}} - 6m_e \right) - \left(m_{\mathrm{B}} - 5m_e \right) - m_e \right) c^2$$
$$= \left(m_{\mathrm{C}} - m_{\mathrm{B}} - 2m_e \right) c^2 .$$

(b)

The value of Q can be found by substituting the values of atomic masses given in the data. We find

$$Q = \left(m_{\mathrm{C}} - m_{\mathrm{B}} - 2m_e \right) c^2$$
$$= \left(11.011433\ \mathrm{u} - 11.009305\ \mathrm{u} - 2 \times 0.0005486\ \mathrm{u} \right) c^2$$
$$= 0.0010308\ \mathrm{u}c^2$$
$$= 0.0010308 \times 931.5\ \mathrm{MeV} = 960.2\ \mathrm{keV}.$$

We note that there is a slight difference between the maximum energy of the positron spectrum 960.8 keV and the value of Q computed above. The difference is

due to the fact that in calculating the nuclear masses from the atomic masses we have not taken into account the difference in binding energy of the electrons before and after the beta decay.