

840.

Problem 54.50 (RHK)

The radionuclide ^{32}P decays to ^{32}S as described by the process $^{32}\text{P} \rightarrow ^{32}\text{S} + e^- + \bar{\nu}$. In a particular decay event, a 1.71-MeV electron is emitted, the maximum possible value. We have to find the kinetic energy of the recoiling ^{32}S atom in this event. The atomic mass of ^{32}S is 31.97 u.



Solution:

The radionuclide ^{32}P decays to ^{32}S as described by the process $^{32}\text{P} \rightarrow ^{32}\text{S} + e^- + \bar{\nu}$. Electron is emitted with maximum possible kinetic energy of 1.71 MeV.

Therefore, in this situation neutrino is emitted with zero kinetic energy and the recoil of the emitted electron is completely taken by the ^{32}S atom.

We will use the relativistic energy relation for determining the momentum of the electron having the kinetic energy of 1.71 MeV.

It is expressed by the equation

$$\sqrt{p^2 c^2 + m_e^2 c^4} = KE + m_e c^2.$$

As $m_e c^2 = 0.511 \text{ MeV}$, and $KE = 1.71 \text{ MeV}$, we have the equation

$$\sqrt{p^2 c^2 + 0.511^2} \text{ MeV} = (1.71 + 0.511) \text{ MeV},$$

or

$$p^2 c^2 = (2.21^2 - 0.511^2) \text{ MeV}^2 = 4.672 \text{ MeV}^2.$$

As the mass of ^{32}S atom is 31.97 u which is very much larger than the electron mass, its speed will be nonrelativistic. The kinetic energy of the ^{32}S atom will therefore be

$$\begin{aligned} &= \frac{p^2}{2m_{^{32}\text{S}}} = \frac{p^2 c^2}{2m_{^{32}\text{S}} c^2} = \frac{4.672 \text{ MeV}^2}{2 \times 31.97 \times 931.5 \text{ MeV}} \\ &= 7.84 \times 10^{-5} \text{ MeV} = 78.4 \text{ eV}. \end{aligned}$$