838.

Problem 54.48 (RHK)

A free neutron decays according to $n \rightarrow p + e^{-} + \bar{\nu}$. *We have to calculate the maximum energy* $K_{\max}$ *of the beta spectrum. Needed atomic masses are*

$$n \ \ 1.008665 \text{ u} \qquad {}^{1}\text{H} \ \ 1.007825 \text{ u} \ .$$

Solution:

A free proton decays according to $n \rightarrow p + e^{-} + \bar{\nu}$.

Electron will carry the maximum energy when momentum of electron is balanced by the momentum of the proton, and the antineutrino, $\bar{\nu}$, is emitted with zero momentum. As the rest mass of $\bar{\nu}$ is zero, its energy will also be zero.

Also as the mass of a proton is 1836 times the mass of an electron, we can use the approximation that its kinetic energy will be negligible compared to the kinetic energy of the beta electron, which will then carry energy nearly equal to the Q value of the decay process

$n \rightarrow p + e^{-} + \bar{\nu}$, which will be

$$
\begin{aligned}
Q &= \left(m(n) - m(p) - m(e)\right)c^2 \\
&= \left(m(n) - m\left({}^{1}\mathrm{H}\right)\right)c^2 \\
&= \left(1.008665 - 1.007825\right)c^2\,\mathbf{u} \\
&= 84 \times 10^{-5}\, c^2\mathbf{u} = 84 \times 10^{-5} \times 931.5\ \mathrm{MeV} \\
&\qquad\qquad = 782.5\ \mathrm{k\,eV}.
\end{aligned}
$$

Thus, we find that

$$K_{\max} = Q = 782.5\ \mathrm{k\,eV}.$$