837.

Problem 54.45 (RHK)

Under certain circumstances, a nucleus can decay by emitting a particle heavier than an α *particle. Such decays are very rare. Consider the decays*

$$^{223}\text{Ra} \rightarrow {}^{209}\text{Pb} + {}^{14}\text{C}$$

and

$$^{223}\text{Ra} \rightarrow {}^{219}\text{Rn} + {}^{4}\text{He}.$$

We have to calculate the Q*-values for these decays and determine that both are energetically possible.* (b) *The Coulomb barrier height for* α *particles in this decay is* 30 MeV. *We have to find the barrier height for* ^{14}C *decay.*

Atomic masses are

^{223}Ra	223.018501 u	^{14}C	14.003242 u
^{209}Pb	208.981065 u	^{4}He	4.002603 u
^{219}Rn	219.009479 u		

Solution:

(a)

We will determine first whether the following reactions are energetically allowed:

$^{223}\text{Ra} \rightarrow {}^{209}\text{Pb} + {}^{14}\text{C}$

and

$^{223}\text{Ra} \rightarrow {}^{219}\text{Rn} + {}^{4}\text{He}.$

We will use the data on values of the atomic masses given in the statement of the problem.

The $Q-$value for the reaction $^{223}\text{Ra} \rightarrow {}^{209}\text{Pb} + {}^{14}\text{C}$ is given by

$$Q = \left(m\left({}^{223}\text{Ra}\right) - m\left({}^{209}\text{Pb}\right) - m\left({}^{14}\text{C}\right)\right)c^2$$

$$= \left(223.018501 - 208.981065 - 14.003242\right)c^2\text{u}$$

$$= 0.034194c^2\text{u} = 0.034194 \times 931.5\ \text{MeV}$$

$$= 31.85\ \text{MeV}.$$

It is positive. Therefore, this reaction is allowed.

The $Q-$value for the reaction $^{223}\text{Ra} \rightarrow {}^{219}\text{Rn} + {}^{4}\text{He}$ is given by

$$Q = \left(m\left({}^{223}\text{Ra}\right) - m\left({}^{219}\text{Rn}\right) - m\left({}^{4}\text{He}\right)\right)c^2$$

$$= \left(223.018501 - 219.009479 - 4.002603\right)c^2\text{u}$$

$$= 0.006419c^2\text{u} = 0.006419 \times 931.5\ \text{MeV}$$

$$= 5.979\ \text{MeV}.$$

It is positive. Therefore, this reaction is allowed.

(b)

The atomic number of ${}^{223}\text{Ra}$ is 88. It is given that the Coulomb barrier height for α particles in this decay is 30 MeV. Let the radius of ${}^{223}\text{Ra}$ nucleus be r m. The Coulomb barrier height for an α particle when it tunnels out of ${}^{223}\text{Ra}$ nucleus is the potential energy between charge 86 e and 2 e separated by r m. That is

$$\frac{86e \times 2e}{4\pi\varepsilon_0 r} = 30\ \text{MeV}.$$

We will use the above result for finding the barrier height for ${}^{14}\text{C}$ decay. As the charge in a ${}^{14}\text{C}$ nucleus is 6 e, the height of the barrier for tunnelling of ${}^{14}\text{C}$ from ${}^{223}\text{Ra}$ nucleus will be

$$\frac{82e \times 6e}{4\pi\varepsilon_0 r} = \frac{30 \times 82 \times 6}{86 \times 2}\ \text{MeV} = 85.8\ \text{MeV}.$$