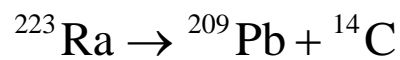


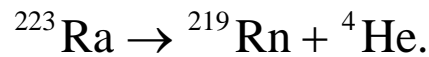
837.

**Problem 54.45 (RHK)**

*Under certain circumstances, a nucleus can decay by emitting a particle heavier than an  $\alpha$  particle. Such decays are very rare. Consider the decays*



and



*We have to calculate the  $Q$ -values for these decays and determine that both are energetically possible. (b) The Coulomb barrier height for  $\alpha$  particles in this decay is 30 MeV. We have to find the barrier height for  ${}^{14}\text{C}$  decay.*

*Atomic masses are*

$${}^{223}\text{Ra} \ 223.018501 \text{ u} \quad {}^{14}\text{C} \ 14.003242 \text{ u}$$

$${}^{209}\text{Pb} \ 208.981065 \text{ u} \quad {}^4\text{He} \ 4.002603 \text{ u}$$

$${}^{219}\text{Rn} \ 219.009479 \text{ u}$$

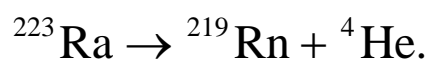
### Solution:

(a)

We will determine first whether the following reactions are energetically allowed:



and



We will use the data on values of the atomic masses given in the statement of the problem.

The  $Q$ -value for the reaction  ${}^{223}\text{Ra} \rightarrow {}^{209}\text{Pb} + {}^{14}\text{C}$  is given by

$$\begin{aligned} Q &= \left( m({}^{223}\text{Ra}) - m({}^{209}\text{Pb}) - m({}^{14}\text{C}) \right) c^2 \\ &= (223.018501 - 208.981065 - 14.003242) c^2 \text{u} \\ &= 0.034194 c^2 \text{u} = 0.034194 \times 931.5 \text{ MeV} \\ &= 31.85 \text{ MeV}. \end{aligned}$$

It is positive. Therefore, this reaction is allowed.

The  $Q$ -value for the reaction  ${}^{223}\text{Ra} \rightarrow {}^{219}\text{Rn} + {}^4\text{He}$  is given by

$$\begin{aligned}
Q &= \left( m(^{223}\text{Ra}) - m(^{219}\text{Rn}) - m(^4\text{He}) \right) c^2 \\
&= (223.018501 - 219.009479 - 4.002603) c^2 \text{u} \\
&= 0.006419 c^2 \text{u} = 0.006419 \times 931.5 \text{ MeV} \\
&= 5.979 \text{ MeV}.
\end{aligned}$$

It is positive. Therefore, this reaction is allowed.

(b)

The atomic number of  $^{223}\text{Ra}$  is 88. It is given that the Coulomb barrier height for  $\alpha$  particles in this decay is 30 MeV. Let the radius of  $^{223}\text{Ra}$  nucleus be  $r$  m. The Coulomb barrier height for an  $\alpha$  particle when it tunnels out of  $^{223}\text{Ra}$  nucleus is the potential energy between charge  $86e$  and  $2e$  separated by  $r$  m. That is

$$\frac{86e \times 2e}{4\pi\epsilon_0 r} = 30 \text{ MeV}.$$

We will use the above result for finding the barrier height for  $^{14}\text{C}$  decay. As the charge in a  $^{14}\text{C}$  nucleus is  $6e$ , the height of the barrier for tunnelling of  $^{14}\text{C}$  from  $^{223}\text{Ra}$  nucleus will be

$$\frac{82e \times 6e}{4\pi\epsilon_0 r} = \frac{30 \times 82 \times 6}{86 \times 2} \text{ MeV} = 85.8 \text{ MeV}.$$

