832.

Problem 54.34 (RHK)

A source contains two phosphorus radionuclides, $^{32}\mathrm{P}$ $\left(t_{1/2} = 14.3\ \mathrm{d}\right)$ *and* $^{33}\mathrm{P}$ $\left(t_{1/2} = 25.3\ \mathrm{d}\right)$. *Initially* 10.0% *of the decays come from* $^{33}\mathrm{P}$. *We have to find the time that one has to wait until* 90.0% *of the decays do so.*

Solution:

A source contains two phosphorus radionuclides, $^{32}\mathrm{P}$ $\left(t_{1/2} = 14.3\ \mathrm{d}\right)$ and $^{33}\mathrm{P}$ $\left(t_{1/2} = 25.3\ \mathrm{d}\right)$. We calculate first the decay constant for $^{32}\mathrm{P}$ and $^{33}\mathrm{P}$ radionuclides.

$$\lambda_{^{32}\mathrm{P}} = \frac{\ln 2}{14.3\ \mathrm{d}} = 4.85 \times 10^{-2}\ \mathrm{d}^{-1},$$

$$\lambda_{^{33}\mathrm{P}} = \frac{\ln 2}{25.3\ \mathrm{d}} = 2.74 \times 10^{-2}\ \mathrm{d}^{-1}\ .$$

Let the initial number of $^{32}\mathrm{P}$ and $^{33}\mathrm{P}$ nuclides be $N_{^{32}\mathrm{P}}$ and $N_{^{33}\mathrm{P}}$, respectively.

It is given that initially 10.0% of decays come from $^{33}\mathrm{P}$. We can express it in the form of an equation as follows:

$$\frac{\lambda_{^{33}\mathrm{P}} N_{^{33}\mathrm{P}}}{\lambda_{^{32}\mathrm{P}} N_{^{32}\mathrm{P}} + \lambda_{^{33}\mathrm{P}} N_{^{33}\mathrm{P}}} = 0.1 \,,$$

or

$$\lambda_{^{33}\mathrm{P}} \left(N_{^{33}\mathrm{P}} / N_{^{32}\mathrm{P}} \right) = 0.1 \lambda_{^{32}\mathrm{P}} + 0.1 \lambda_{^{33}\mathrm{P}} \left(N_{^{33}\mathrm{P}} / N_{^{32}\mathrm{P}} \right),$$

or

$$\lambda_{^{33}\mathrm{P}} N_{^{33}\mathrm{P}} / \lambda_{^{32}\mathrm{P}} N_{^{32}\mathrm{P}} = 1/9 \,.$$

Let the time for which one has to wait until 90.0% of the decays come from $^{33}\mathrm{P}$ be t d.

We can express it as an equation using the radioactive decay law as follows:

$$\frac{\lambda_{^{33}\mathrm{P}} N_{^{33}\mathrm{P}} e^{-\lambda_{^{33}\mathrm{P}} t}}{\lambda_{^{32}\mathrm{P}} N_{^{32}\mathrm{P}} e^{-\lambda_{^{32}\mathrm{P}} t} + \lambda_{^{33}\mathrm{P}} N_{^{33}\mathrm{P}} e^{-\lambda_{^{33}\mathrm{P}} t}} = 0.9 \,,$$

or

$$\lambda_{^{33}\mathrm{P}} e^{-\left(\lambda_{^{33}\mathrm{P}} - \lambda_{^{32}\mathrm{P}}\right)t} \left(N_{^{33}\mathrm{P}} / N_{^{32}\mathrm{P}} \right) = 0.9 \lambda_{^{32}\mathrm{P}} + 0.9 \lambda_{^{33}\mathrm{P}} e^{-\left(\lambda_{^{33}\mathrm{P}} - \lambda_{^{32}\mathrm{P}}\right)t} \left(N_{^{33}\mathrm{P}} / N_{^{32}\mathrm{P}} \right),$$

or

$$0.1 \times e^{-\left(\lambda_{^{33}\mathrm{P}} - \lambda_{^{32}\mathrm{P}}\right)t} = \frac{0.9 \lambda_{^{32}\mathrm{P}}}{\lambda_{^{33}\mathrm{P}} \times \left(N_{^{33}\mathrm{P}} / N_{^{32}\mathrm{P}} \right)},$$

or

$$e^{-\left(\lambda_{^{33}\mathrm{P}}-\lambda_{^{32}\mathrm{P}}\right)t}=9\times 9,$$

or

$$e^{(4.85\text{-}2.74)\times 10^{-2}\times t}=81,$$

or

$$\left(4.85\text{-}2.74\right)\times 10^{-2}\times t=4.39,$$

or

$$t=208.$$

That is for decays from $^{33}\mathrm{P}$ nuclide to be 90.0% of the decays of the two nuclides one has to wait for 208 days, if initially it was 10.0%.