

832.

**Problem 54.34 (RHK)**

*A source contains two phosphorus radionuclides,  $^{32}\text{P}$  ( $t_{1/2} = 14.3 \text{ d}$ ) and  $^{33}\text{P}$  ( $t_{1/2} = 25.3 \text{ d}$ ). Initially 10.0% of the decays come from  $^{33}\text{P}$ . We have to find the time that one has to wait until 90.0% of the decays do so.*

**Solution:**

A source contains two phosphorus radionuclides,  $^{32}\text{P}$  ( $t_{1/2} = 14.3 \text{ d}$ ) and  $^{33}\text{P}$  ( $t_{1/2} = 25.3 \text{ d}$ ). We calculate first the decay constant for  $^{32}\text{P}$  and  $^{33}\text{P}$  radionuclides.

$$\lambda_{^{32}\text{P}} = \frac{\ln 2}{14.3 \text{ d}} = 4.85 \times 10^{-2} \text{ d}^{-1},$$

$$\lambda_{^{33}\text{P}} = \frac{\ln 2}{25.3 \text{ d}} = 2.74 \times 10^{-2} \text{ d}^{-1}.$$

Let the initial number of  $^{32}\text{P}$  and  $^{33}\text{P}$  nuclides be  $N_{^{32}\text{P}}$  and  $N_{^{33}\text{P}}$ , respectively.

It is given that initially 10.0% of decays come from  $^{33}\text{P}$ .

We can express it in the form of an equation as follows:

$$\frac{\lambda_{33\text{P}} N_{33\text{P}}}{\lambda_{32\text{P}} N_{32\text{P}} + \lambda_{33\text{P}} N_{33\text{P}}} = 0.1 ,$$

or

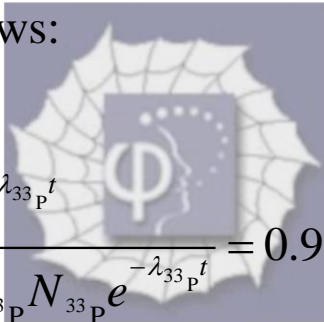
$$\lambda_{33\text{P}} \left( N_{33\text{P}} / N_{32\text{P}} \right) = 0.1 \lambda_{32\text{P}} + 0.1 \lambda_{33\text{P}} \left( N_{33\text{P}} / N_{32\text{P}} \right),$$

or

$$\lambda_{33\text{P}} N_{33\text{P}} / \lambda_{32\text{P}} N_{32\text{P}} = 1/9 .$$

Let the time for which one has to wait until 90.0% of the decays come from  $^{33}\text{P}$  be  $t$  d.

We can express it as an equation using the radioactive decay law as follows:



$$\frac{\lambda_{33\text{P}} N_{33\text{P}} e^{-\lambda_{33\text{P}} t}}{\lambda_{32\text{P}} N_{32\text{P}} e^{-\lambda_{32\text{P}} t} + \lambda_{33\text{P}} N_{33\text{P}} e^{-\lambda_{33\text{P}} t}} = 0.9 ,$$

or

$$\lambda_{33\text{P}} e^{-(\lambda_{33\text{P}} - \lambda_{32\text{P}})t} \left( N_{33\text{P}} / N_{32\text{P}} \right) = 0.9 \lambda_{32\text{P}} + 0.9 \lambda_{33\text{P}} e^{-(\lambda_{33\text{P}} - \lambda_{32\text{P}})t} \left( N_{33\text{P}} / N_{32\text{P}} \right),$$

or

$$0.1 \times e^{-(\lambda_{33\text{P}} - \lambda_{32\text{P}})t} = \frac{0.9 \lambda_{32\text{P}}}{\lambda_{33\text{P}} \times \left( N_{33\text{P}} / N_{32\text{P}} \right)},$$

or

$$e^{-(\lambda_{^{33}\text{P}} - \lambda_{^{32}\text{P}})t} = 9 \times 9,$$

or

$$e^{(4.85-2.74) \times 10^{-2} \times t} = 81,$$

or

$$(4.85-2.74) \times 10^{-2} \times t = 4.39,$$

or

$$t = 208.$$

That is for decays from  $^{33}\text{P}$  nuclide to be 90.0% of the decays of the two nuclides one has to wait for 208 days, if initially it was 10.0%.

