828.

Problem 54.25 (RHK)

A radioactive isotope of mercury, $^{197}\mathrm{Hg}$, *decays into gold,* $^{197}\mathrm{Au}$, *with a decay constant of* $0.0108\ \mathrm{h}^{-1}$. (a) *We have to calculate its half-life,* (b) *the fraction of the original amount that will remain after three half-lives, after* 10 *days.*

Solution:

(a)

A radioactive isotope of mercury, $^{197}\mathrm{Hg}$, decays into gold, $^{197}\mathrm{Au}$, with a decay constant of $0.0108\ \mathrm{h}^{-1}$. Half-life and decay constants by definition are related as

$$\frac{1}{2} = e^{-\lambda t_{1/2}}.$$

$$\therefore\ \lambda t_{1/2} = \ln 2,$$

and

$$t_{1/2} = \frac{\ln 2}{\lambda}.$$

As $\lambda = 0.0108\ \mathrm{h}^{-1}$,

$$t_{1/2} = \frac{\ln 2}{0.0108}\ \mathrm{h} = 64.18\ \mathrm{h}.$$

(b)

The fraction of the original amount that will remain after three half-lives will be

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^3 = 0.125.$$

And, the fraction of the original amount that will remain after 10 days will be

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/t_{1/2}} = \left(\frac{1}{2}\right)^{240/64.18} = \left(\frac{1}{2}\right)^{3.739} = 0.0749.$$