826.

Problem 54.22 (RHK)

We have to show (a) *that the electrostatic potential energy of a uniform sphere of charge Q and radius R is given by*

$$U = \frac{3Q^2}{20\pi\varepsilon_0 R} \ .$$

(b) *We have to find the electrostatic potential energy for the nuclide* $^{239}\mathrm{Pu}$, *assumed spherical.* (c) *We have to compare its electrostatic potential energy per particle with its binding energy per nucleon of* 7.56 MeV. (d) *What do we conclude?*

Solution:

(a)

Let ρ be the charge density in a sphere of radius *R*, which contains charge Q distributed uniformly inside it. We have

$\rho = Q\big/\left(4\pi R^3/3\right).$

We will build uniformly charged sphere by adding thin spherical shells containing charge of density ρ by bringing them from infinity so that step by step the radius of the sphere increase from r to $r+\Delta r$. The change in potential energy of the sphere in such a process will be given by

$$\Delta U(r) = \frac{\left(4\pi r^3 \rho/3\right)\times\left(4\pi r^2 \Delta r \rho\right)}{4\pi\varepsilon_0 r}$$

$$= \frac{4\pi\rho^2}{3\varepsilon_0}\times r^4 \Delta r \; .$$

Therefore, the electrostatic potential energy of a sphere of radius R containing charge Q distributed uniformly inside it will be

$$U = \int_0^R \Delta U(r) = \int_0^R \frac{4\pi\rho^2}{3\varepsilon_0} r^4 dr$$

$$= \frac{4\pi\rho^2 R^5}{3\varepsilon_0 \times 5} = \frac{4\pi R^5}{15\varepsilon_0}\left(\frac{3Q}{4\pi R^3}\right)^2$$

$$= \frac{3Q^2}{20\pi\varepsilon_0 R} \; .$$

(b)

We will use this result for calculating the potential energy of the nuclide $^{239}\mathrm{Pu}$. The radius of the nuclide is

$R = R_0 A^{1/3} = 1.2 \times (239)^{1/3}$ fm $= 7.45$ fm.

The atomic number of $^{239}\mathrm{Pu}$ nuclide is 94. Therefore, $Q = 94e$. The electrostatic potential energy of $^{239}\mathrm{Pu}$ will therefore be

$$U\left({}^{239}\mathrm{Pu}\right) = \frac{3 \times \left(94 \times 1.6 \times 10^{-19}\right)^2 \times 8.99 \times 10^9}{5 \times 7.45 \times 10^{-15}} \text{ J},$$

$$= 16.37 \times 10^{-11} \text{ J}$$

$$= \frac{16.37 \times 10^{-11}}{1.6 \times 10^{-13}} \text{ MeV} = 10.23 \times 10^2 \text{ MeV}.$$

In carrying out the above calculation, we have used

$$\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}.$$

Therefore, the electrostatic potential energy per nucleon in the nuclide $^{239}\mathrm{Pu}$ will be

$$\left(10.23 \times 10^2 / 239\right) \text{ MeV} = 4.28 \text{ MeV},$$

And the electrostatic potential energy per proton will be

$$\left(10.23 \times 10^2 / 94\right) \text{ MeV} = 10.88 \text{ MeV}.$$

The binding energy per nucleon in a $^{239}\mathrm{Pu}$ nuclide is 7.56 MeV.

(d)

We conclude that as we move to larger and larger nuclei the component of electrostatic potential energy increases very rapidly and therefore for nucleons to be bound the number of neutrons necessarily has to be more than the number of protons in nuclides of large mass number.