

823.

Problem 54.14 (RHK)

Because a nucleon is confined to a nucleus, we can take its uncertainty in position to be the approximately the nuclear radius R . Using the uncertainty principle, we have to estimate the kinetic energy of a nucleon in a nucleus with, say $A=100$. We may take the uncertainty in momentum Δp to be the actual momentum p .

Solution:



For estimating the radius of a nucleus of mass number $A = 100$, we will use the empirical relation

$$R = R_0 A^{1/3}, \quad R_0 = 1.2 \text{ fm} = 1.2 \times 10^{-15} \text{ m}.$$

Therefore, the radius R of a nucleus of mass number 100 will be

$$R = 1.2 \times 10^{-15} \times (100)^{1/3} \text{ m} = 5.57 \times 10^{-15} \text{ m}.$$

The Heisenberg uncertainty relation is

$$\Delta p \Delta x = h.$$

We take uncertainty in position to be approximately the nuclear radius R , i.e.

$$\Delta x = R = 5.57 \times 10^{-15} \text{ m},$$

and uncertainty in momentum to be the momentum of the nucleon, i.e.

$$\Delta p = p.$$

Using the uncertainty relation, we find

$$\begin{aligned} p &= \frac{h}{R} = \frac{1.05 \times 10^{-34}}{5.57 \times 10^{-15}} \text{ kg m s}^{-1} \\ &= 1.88 \times 10^{-20} \text{ kg m s}^{-1}. \end{aligned}$$

For the mass of a nucleon we take the mass of proton

$$m = 1.672 \times 10^{-27} \text{ kg}.$$

Our estimate of the kinetic energy of a nucleon in a nucleus of mass number 100 is

$$\begin{aligned} KE_{\text{nucleon}} &= \frac{p^2}{2m} = \frac{(1.88 \times 10^{-20})^2}{2 \times 1.672 \times 10^{-27}} \text{ J} \\ &= 1.06 \times 10^{-13} \text{ J} = \frac{1.06 \times 10^{-13}}{1.6 \times 10^{-13}} \text{ MeV} \\ &= 6.64 \times 10^{-1} \text{ MeV}. \end{aligned}$$