820.

Problem 54.4 (RHK)

When an α *particle collides elastically with a nucleus, the nucleus recoils. A* 5.00-MeV α *particle has a head-on elastic collision with a gold nucleus, initially at rest. We have to find the kinetic energy (a) of the recoiling nucleus and* (b) *of the rebounding* α *particle. The mass of the* α *particle may be taken to be* 4.00 u *and that of the gold nucleus* 197 u.

$1\text{ u} = 1.66\times10^{-27}\text{ kg}$.

Solution:

We will use principles of conservation of energy and momentum for answering this problem. Let *M* be the mass of gold nucleus and *m* be the mass of α particle. Let *E* be the kinetic energy of the incident α particle, and let *P* be the momentum of the recoiling gold nucleus and let *p* be the momentum of the rebounding α particle. As the collision is elastic we will require that the sum of the kinetic energies of the recoiling gold nucleus and the rebounding α particle be equal to the kinetic energy of the incident α particle.

Momentum of the incident α particle, $p_i = (2mE)^{1/2}$.

From the conservation of momentum we get the following equation:

$$(2mE)^{1/2} = P - p.$$

And from the conservation of energy, we get the following equation:

$$\frac{P^2}{2M} + \frac{p^2}{2m} = E.$$

From the above two equations, after algebraic simplification, we get

$$P\left((m+M)P - 2M(2mE)^{1/2}\right) = 0.$$

There are two solutions for P. The first one $P = 0$ implies that the gold nucleus continues to remain at rest after collision with the α particle, which contradicts the statement of the problem. We therefore select the second solution

$$P = \frac{2M}{m+M}(2mE)^{1/2}.$$

We substitute the data:

$m = 4.0$ u,

$M = 197$ u,

and

$E = 5.00 \text{ MeV} = 5.0 \times 1.6 \times 10^{-13}$ J.

We find

$$P = \frac{2 \times 197}{4 + 197} \times \left(2 \times 4 \times 1.66 \times 10^{-27} \times 5.0 \times 1.6 \times 10^{-13}\right)^{1/2} \text{ kg m s}^{-1}$$

$$= 20.19 \times 10^{-20} \text{ kg m s}^{-1}.$$

The kinetic energy of the recoiling gold nucleus will therefore be

$$KE_{\text{gold nucleus}} = \frac{P^2}{2M} = \frac{\left(20.19 \times 10^{-20}\right)^2}{2 \times 197 \times 1.66 \times 10^{-27}} \text{ J}$$

$$= 6.23 \times 10^{-14} \text{ J}$$

$$= \frac{6.23 \times 10^{-14}}{1.6 \times 10^{-13}} \text{ MeV} = 0.39 \text{ Mev.}$$

From conservation of energy, we note that the kinetic energy of the rebounding α particle will be

$$KE_{\text{rebounding}\,\alpha\text{ particle}} = \left(5.0 - 0.39\right) \text{ MeV} = 4.61 \text{ MeV.}$$