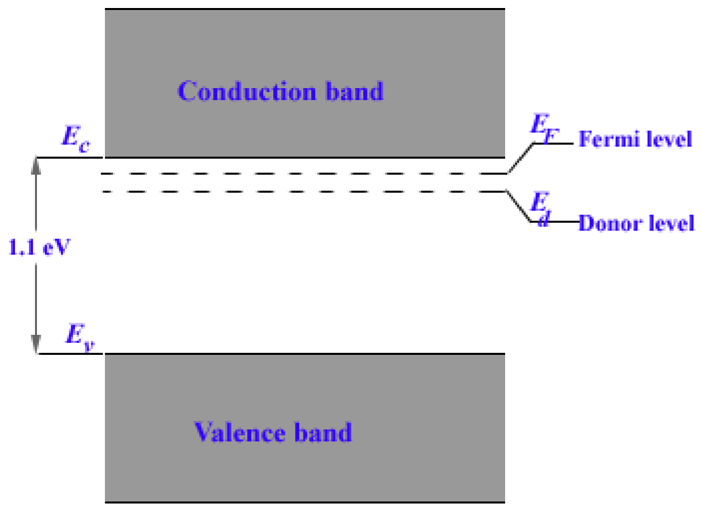
812.

Problem 53.41 (RHK)

A silicon sample is doped with atoms having a donor state 0.11 eV *below the bottom of the conduction band. If each of these states is occupied with probability* 4.8×10^{-5} *at* 290 K, *we have to find the Fermi level relative to the top of the valence band.* (b) *We have to calculate the probability that a state at the bottom of the conduction band is occupied. The energy gap in silicon is* 1.1 eV.

Solution:



(a)

We have been given that the donor state lies 0.11 eV below the bottom of the conduction band. That is

$$E_c - E_d = 0.11 \text{ eV}.$$

Let

$$E_c - E_F = \mathrm{E} \text{ eV}.$$

We have

$$E_d - E_F = \left(\mathrm{E} - 0.11\right) \text{ eV}.$$

It is given that the probability that a donor state to be occupied at 290 K is 4.8×10^{-5}.

Therefore,

$$\frac{1}{e^{\left(E_d - E_F\right)/kT} + 1} = \frac{1}{e^{\left(\mathrm{E}-0.11\right)\text{ eV}/kT} + 1} = 4.8\times10^{-5}.$$

We note that

$$kT = 8.62\times10^{-5}\times290 \text{ eV} = 2.5\times10^{-2} \text{ eV}.$$

We thus have the equation

$$\frac{1}{e^{(E_d - E_F)/kT} + 1} = \frac{1}{e^{(\mathrm{E}-0.11)\ \mathrm{eV}/0.025\ \mathrm{eV}} + 1} = 4.8\times10^{-5},$$

or

$$e^{(\mathrm{E}-0.11)\ /0.025} + 1 = \left(1/4.8\times10^{-5}\right)$$

$$= 20833,$$

or

$$e^{(\mathrm{E}-0.11)\ /0.025} = 20832,$$

or

$$\left(\mathrm{E}-0.11\right)\ /0.025 = 9.944,$$

or

$$\mathrm{E} = 9.944\times0.025+0.11 = 0.358.$$

Therefore, relative to the top of the valence band the Fermi level will be at

$$E_F - E_v = -\left(E_c - E_F\right) + \left(E_c - E_v\right)$$

$$= -0.358\ \mathrm{eV} + 1.1\ \mathrm{eV} = 0.742\ \mathrm{eV}.$$

The Fermi level is at $0.742\ \mathrm{eV}$ above the top of the valence band.

(b)

We calculate next the probability that a state at the bottom of the conduction band will be occupied. It is given by

$$p\left(E_c\right)=\frac{1}{e^{\left(E_c-E_F\right)/kT}+1}=\frac{1}{\exp\left(0.358/0.025\right)+1}$$

$$=\frac{1}{e^{14.32}+1}\;;\;\; e^{-14.32}=6.0\times10^{-7}.$$