811.

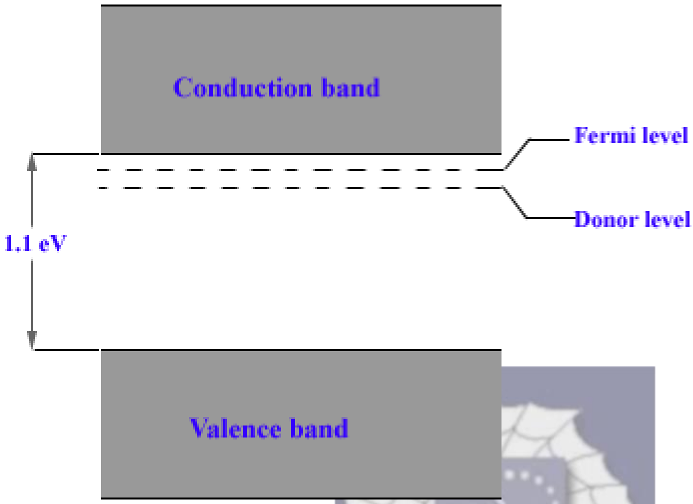
<u>Problem 53.40 (RHK)</u>

Doping changes the Fermi energy of a semiconductor. Consider silicon, with a gap of 1.1 eV *between the valence and conduction bands. At* 290 K *the Fermi level of the pure material is nearly at the midpoint of the gap. Suppose that it is doped with donor atoms, each of which has a state* 0.15 eV *below the bottom of the conduction band, and suppose further that the doping raises the Fermi level to* 0.084 eV *below the bottom of that band.* (a) *We have to calculate both for pure and doped silicon the probability that a state at the bottom of the conduction band is occupied.* (b) *We have to also calculate that a donor state in the doped material is occupied.*

Solution:

(a)

We will consider first the case of *pure silicon*. It is given that for pure silicon the Fermi level is near the midpoint of the gap. Therefore, for a state at the bottom of the conduction band, we have $E - E_F = 0.55$ eV. We are asked to calculate the probability for pure silicon at 290 k that a state at the bottom of the conduction band is occupied. Using the Fermi-Dirac distribution function we will calculate the probability for occupation of the state which is at the bottom of the conduction band.



$$p(E) = \frac{1}{e^{(E-E_F)/kT} + 1}.$$

$$\frac{(E - E_F)}{kT} = \frac{0.55 \text{ eV}}{8.62 \times 10^{-5} \times 290 \text{ eV}} = 22.0.$$

Therefore,

$$p_{\text{pure silicon}}(E) = \frac{1}{e^{22}+1} \approx e^{-22} = 2.79\times10^{-10}.$$

We will now calculate the probability for occupation of the level at the bottom of conduction band for the doped silicon. It is given that the doping raises the Fermi level and that the Fermi level is 0.84 eV below the bottom of the conduction band.

We find

$$\frac{(E-E_F)}{kT} = \frac{0.084\text{ eV}}{8.62\times10^{-5}\times290\text{ eV}} = 3.36.$$

Therefore,

$$p_{\text{doped silicon}}(E) = \frac{1}{e^{3.36}+1} = \frac{1}{28.8+1} = 0.034\ ,$$

which is (3.4%).

(b)

We calculate the probability that a donor state will be occupied. It is given that donor states lie about 0.15 eV below the bottom of the conduction band.

As $(E_c - E_d) = 0.15$ eV, and $(E_c - E_F) = 0.084$ eV,

we have

$$E_d - E_F = -(0.15-0.084)\text{ eV} = -0.066\text{ eV}.$$

And

$$\frac{E_d - E_F}{kT} = -\frac{0.066 \text{ eV}}{8.62 \times 10^{-5} \times 290 \text{ eV}} = -2.64 \; .$$

Therefore, the probability for a donor state to be occupied will be

$$p_{\text{donor state}} = \frac{1}{e^{-2.64} + 1} = \frac{1}{0.07 + 1} = 0.93 \; . \; (93\%)$$