

807.

Problem 53.33 (RHK)

In a simplified model of an intrinsic semiconductor (no doping), the actual distribution in energy of states is replaced by one in which there are N_v states in the valence band, all these states having the same energy E_v , and N_c states in the conduction band, all these states having the same energy E_c . The number of electrons in the conduction band equals the number of holes in the valence band. (a) We have to show that this last condition implies that



$$\frac{N_c}{e^{(E_c - E_F)/kT} + 1} = \frac{N_v}{e^{-(E_v - E_F)/kT} + 1} .$$

(b) If the Fermi level is in the gap between the two bands and is far from both ends compared to kT , then the exponentials dominate in the denominators. Under these conditions, we have to show that

$$E_F = \frac{1}{2}(E_c + E_v) + \frac{1}{2}kT \ln(N_v/N_c) ,$$

and therefore that, if $N_v \approx N_c$, the Fermi level is close to the centre of the gap.

Solution:

(a)

In the given model of an intrinsic semiconductor it is assumed that the actual distribution in energy of states is replaced by one in which there are N_v states in the valence band, all these states having the same energy E_v , and N_c states in the conduction band, all these states having the same energy E_c .

The probability of occupation of a state with energy E_c is therefore

$$p(E_c) = \frac{1}{e^{(E_c - E_F)/kT} + 1}$$



Therefore, the number of electrons in the conduction band will be given by

$$p(E_c)N_c = \frac{N_c}{e^{(E_c - E_F)/kT} + 1} .$$

The probability of finding a hole at energy E_v is

$1 - p(E_v)$, which is

$$\frac{1}{e^{-(E_v - E_F)/kT} + 1} .$$

Therefore, the number of holes in the valence band will be given by

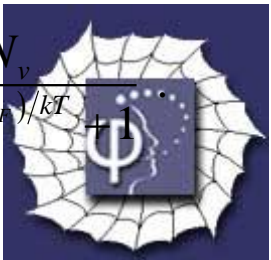
$N_v p_h(E_v)$, which is

$$\frac{N_v}{e^{-(E_v - E_F)/kT} + 1}.$$

We require that the number of electrons in the conduction band equals the number of holes in the valence band. This condition implies that

$$p(E_c) N_c = N_v p_h(E_v),$$

or

$$\frac{N_c}{e^{(E_c - E_F)/kT} + 1} = \frac{N_v}{e^{-(E_v - E_F)/kT} + 1}.$$


(b)

We assume that the Fermi level is in the gap between the two bands and is far from both ends compared to kT , and thus the exponentials dominate in the denominators.

Under this assumption, we have

$$\frac{N_c}{e^{(E_c - E_F)/kT}} = \frac{N_v}{e^{-(E_v - E_F)/kT}},$$

or

$$\frac{N_c}{N_v} = \frac{e^{(E_c - E_F)/kT}}{e^{(E_F - E_v)/kT}}.$$

Taking \ln of both sides of the above equation, we get

$$\frac{(E_c - E_F)}{kT} + \frac{(E_v - E_F)}{kT} = \ln\left(\frac{N_c}{N_v}\right),$$

or

$$E_F = \frac{1}{2}(E_c + E_v) + \frac{1}{2}kT \ln(N_v/N_c).$$

Therefore, if $N_v \approx N_c$,

$$\ln(N_v/N_c) = 0 \text{ and}$$

$$E_F = \frac{1}{2}(E_c + E_v),$$

that is the Fermi level is close to the centre of the gap between the conduction band and the valence band.

