807.

Problem 53.33 (RHK)

In a simplified model of an intrinsic semiconductor (*no doping*), *the actual distribution in energy of states is replaced by one in which there are* N_v *states in the valence band, all these states having the same energy* E_v, *and* N_c *states in the conduction band, all these states having the same energy* E_c. *The number of electrons in the conduction band equals the number of holes in the valence band.* (a) *We have to show that this last condition implies that*

$$\frac{N_c}{e^{(E_c-E_F)/kT}+1}=\frac{N_v}{e^{-(E_v-E_F)/kT}+1} .$$

(b) *If the Fermi level is in the gap between the two bands and is far from both ends compared to* kT, *then the exponentials dominate in the denominators. Under these conditions, we have to show that*

$$E_F=\tfrac{1}{2}\left(E_c+E_v\right)+\tfrac{1}{2}kT\ln\left(N_v/N_c\right) ,$$

and therefore that, if $N_v \approx N_c$, *the Fermi level is close to the centre of the gap.*

Solution:

(a)

In the given model of an intrinsic semiconductor it is assumed that the actual distribution in energy of states is replaced by one in which there are N_v states in the valence band, all these states having the same energy E_v, and N_c states in the conduction band, all these states having the same energy E_c.

The probability of occupation of a state with energy E_c is therefore

$$p(E_c) = \frac{1}{e^{(E_c - E_F)/kT} + 1}.$$

Therefore, the number of electrons in the conduction band will be given by

$$p(E_c)N_c = \frac{N_c}{e^{(E_c - E_F)/kT} + 1} \ .$$

The probability of finding a hole at energy E_v is $1 - p(E_v)$, which is

$$\frac{1}{e^{-(E_v - E_F)/kT} + 1} \ .$$

Therefore, the number of holes in the valence band will be given by

$N_v p_h\left(E_v\right)$, which is

$$\frac{N_v}{e^{-(E_v-E_F)/kT}+1}\ .$$

We require that the number of electrons in the conduction band equals the number of holes in the valence band. This condition implies that

$$p\left(E_c\right)N_c = N_v p_h\left(E_v\right),$$

or

$$\frac{N_c}{e^{(E_c-E_F)/kT}+1} = \frac{N_v}{e^{-(E_v-E_F)/kT}+1}.$$

(b)

We assume that the Fermi level is in the gap between the two bands and is far from both ends compared to kT, and thus the exponentials dominate in the denominators. Under this assumption, we have

$$\frac{N_c}{e^{(E_c-E_F)/kT}} = \frac{N_v}{e^{-(E_v-E_F)/kT}}\ ,$$

or

$$\frac{N_c}{N_v} = \frac{e^{(E_c-E_F)/kT}}{e^{(E_F-E_v)/kT}}.$$

Taking *ln* of both sides of the above equation, we get

$$\frac{(E_c - E_F)}{kT} + \frac{(E_v - E_F)}{kT} = \ln\left(\frac{N_c}{N_v}\right),$$

or

$$E_F = \tfrac{1}{2}(E_c + E_v) + \tfrac{1}{2}kT\ln(N_v/N_c).$$

Therefore, if $N_v \approx N_c$,

$\ln(N_v/N_c) = 0$ and

$$E_F = \tfrac{1}{2}(E_c + E_v),$$

that is the Fermi level is close to the centre of the gap between the conduction band and the valence band.