805.

Problem 53.29 (RHK)

For silver, we have to calculate (a) *the mean free path of conduction electrons and* (b) *the ratio of mean free path to the distance between neighbouring ions. Silver has a Fermi energy of* 5.51 eV *and a resistivity of* $1.62\times10^{-8}\ \Omega\,\text{m}$.

Solution:

Resistivity of a metal is given in terms of the density of conduction electrons, n, the average time of collision of electrons with the lattice, τ, and the mass of electron, m, by the relation

$$\rho_{\text{resistivity}} = \frac{m}{ne^2\tau},$$

or

$$\tau = \frac{m}{ne^2\rho_{\text{resistivity}}}.$$

In problem **804** $\left(\text{Problem 53.29 (RHK)}\right)$, we have calculated that the number of conduction electrons per cubic meter in silver metal is

$n = 5.86 \times 10^{28}\ \text{m}^{-3}.$

Therefore, the mean time for collision of a conduction electron with a silver ion at the lattice will be

$$\tau = \frac{9.11 \times 10^{-31}\ \text{kg}}{5.86 \times 10^{28}\ \text{m}^{-3} \times \left(1.6 \times 10^{-19}\ \text{C}\right)^2 \times 1.62 \times 10^{-8}\ \Omega\ \text{m}}$$

$$= 0.37 \times 10^{-13}\ \text{s},$$

where we have used that

$\rho_{\text{resistivity}} = 1.62 \times 10^{-8}\ \Omega\ \text{m}.$

As conduction electrons will have speed of about $v_F = 1.39 \times 10^{6}\ \text{m s}^{-1}$ (see problem 804), the mean free path for conduction electrons will be

$$v_F \tau = 1.39 \times 10^{6} \times 0.37 \times 10^{-13}\ \text{m}$$
$$= 0.514 \times 10^{-7}\ \text{m} = 51.4\ \text{nm}.$$

We calculate next the distance between neighbouring ion cores. Let us assume that each ion core is contained inside a cubical lattice of length a, the distance between neighbouring ion cores. We thus have the relation

$a^3 \times n = 1\ \text{m}^3.$

We find

$$a = \left(\frac{1}{n}\right)^{1/3} = \left(\frac{1}{5.86 \times 10^{28}}\right)^{1/3}\ \text{m} = 2.58 \times 10^{-10}\ \text{m}.$$

We estimate the ratio of the mean free path to the distance between neighbouring ion cores. It is given by

$$\frac{\text{mean free path of conduction electrons}}{a} = \frac{0.514 \times 10^{-7}}{2.58 \times 10^{-10}}$$

$$= 199 \; .$$