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Problem 53.27 (RHK)

Silver is a monovalent metal. We have to calculate (a) *the number of conduction electrons per cubic meter*, (b) *the Fermi energy*, (c) *the Fermi speed, and* (d) *de Broglie wavelength corresponding to this speed*.

Solution:

(a)

From the physical data for silver, we note that

molar mass of silver, $M_{\text{Ag}} = 107.68 \text{ g mol}^{-1}$,

density of silver, $\rho_{\text{Ag}} = 10.49 \text{ g cm}^{-3}$.

Therefore, the number of silver atoms per cubic meter will be

$$n = \frac{N_A \rho_{\text{Ag}}}{M_{\text{Ag}}} = \frac{6.02\times10^{23} \text{ mol}^{-1} \times 10.49\times10^{3} \text{ kg m}^{-3}}{107.68\times10^{-3} \text{ kg mol}}$$

$$= 5.86\times10^{28} \text{m}^{-3}.$$

As silver is a monovalent metal, each atom of silver contributes one conduction electron. Therefore, the number of conduction electrons per cubic meter of silver will be equal to the number of silver atoms per cubic meter in metallic silver.

(b)

We calculate next the Fermi energy for the conduction electrons in metallic silver using the relation

$$E_F = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3}.$$

Using that for silver $n = 5.86\times10^{28}\,\mathrm{m}^{-3}$, we find that for conduction electrons of silver the Fermi energy will be

$$\begin{aligned} E_F = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3} &= \frac{\left(6.63\times10^{-34}\right)^2}{8\times9.11\times10^{-31}}\times\left(\frac{3\times5.86\times10^{28}}{\pi}\right)^{2/3}\ \mathrm{J} \\ &= 8.822\times10^{-19}\ \mathrm{J} \\ &= 8.822\times10^{-19}\times6.242\times10^{18}\ \mathrm{eV} \\ &= 5.51\ \mathrm{eV}. \end{aligned}$$

(c)

We calculate next the Fermi speed for conduction electrons in silver metal. We have

$$\frac{1}{2}mv_F^2 = E_F,$$

or

$$\begin{aligned} v_F = \left(\frac{2E_F}{m}\right)^{1/2} &= \left(\frac{2\times8.822\times10^{-19}\ \mathrm{J}}{9.11\times10^{-31}\ \mathrm{kg}}\right)^{1/2} \\ &= 1.39\times10^{6}\ \mathrm{m\,s^{-1}}. \end{aligned}$$

(d)

The de Broglie wavelength of conduction electrons moving with Fermi speed v_F will be

$$\lambda_F = \frac{h}{mv_F} = \frac{6.63\times10^{-34}}{9.11\times10^{-31}\times1.39\times10^{6}}\ \text{m}$$

$$= 5.236\times10^{-10}\ \text{m} = 523.6\ \text{pm}.$$