**803.**

**<u>Problem 53.25 (RHK)</u>**

*We have to show that at absolute zero of temperature the average energy* $\bar{E}$ *of the conduction electrons in a metal is equal to* $\frac{3}{5}E_F$*, where* $E_F$ *is the Fermi energy.*

**<u>Solution:</u>**

For a gas of electrons the average energy is defined by the relation

$$\bar{E} = \frac{1}{n}\int_0^\infty E n_o(E)dE \ ,$$

where $n_o\left(E\right)dE$ is the number of electrons per unit volume having energy between $E$ and $E+dE$. At absolute zero of temperature the probability of occupation is given by the function

$$p\left(E\right) = 1 \text{ for } E < E_F,$$
$$= 0 \text{ for } E > E_F \ .$$

As the number density of electrons having energy between $E$ and $E+dE$ is

$$n(E)\,dE = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}\sqrt{E}\,dE,$$

and $n_o(E)\,dE = n(E)\,dE \times p(E)$,

we have

$$\bar{E} = \frac{1}{n}\int_0^\infty E n_o(E)\,dE = \frac{1}{n}\int_0^{E_F} \frac{8\sqrt{2}\pi m^{3/2} E^{3/2}}{h^3}\,dE$$

$$= \frac{8\sqrt{2}\pi m^{3/2}}{nh^3}\,\frac{E_F^{5/2}}{5/2}.$$

The number of electrons per unit volume at absolute zero of temperature is given by

$$n = \int_0^{E_F} \frac{8\sqrt{2}\pi m^{3/2} E^{1/2}}{h^3}\,dE$$

$$= \frac{8\sqrt{2}\pi m^{3/2}}{h^3}\,\frac{E_F^{3/2}}{3/2},$$

therefore,

$$\bar{E} = \frac{1}{n}\int_0^\infty E n_o(E)\,dE = \frac{3}{5}E_F\ .$$