

802.

Problem 53.20 (RHK)

A neutron star can be analyzed by techniques similar to those used for ordinary metals. In this case the neutrons (rather than electrons) obey the Fermi-Dirac probability function. We have to consider a neutron star of 2.00 solar masses with a radius of 10.0 km and calculate the Fermi energy of the neutrons.

Solution:

Mass of a neutron, $m_n = 1.67 \times 10^{-27}$ kg,

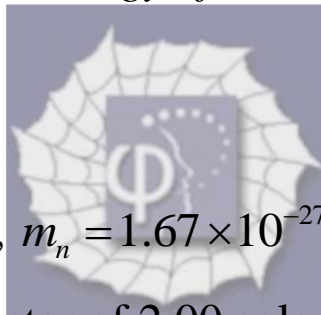
Mass of a neutron star of 2.00 solar masses,

$$M = 2 \times 1.99 \times 10^{30} \text{ kg},$$

Radius of the neutron star, $R = 10.0 \times 10^3$ m.

Number of neutrons in the neutron star,

$$N = \frac{2 \times 1.99 \times 10^{30}}{1.67 \times 10^{-27}} = 2.383 \times 10^{57}.$$



From the above data we note that the number of neutron per cubic meter in the neutron star,

$$n = \frac{N}{\left(4\pi R^3/3\right)} = \frac{2.383 \times 10^{57}}{\left(4\pi (10^4)^3/3\right)} \text{ neutrons per m}^3$$

$$= 5.688 \times 10^{44} \text{ neutrons per m}^3.$$

At absolute zero Fermi energy is the highest energy of the occupied state. It is given by the relation

$$E_F = \frac{h^2}{8m} \left(\frac{3n}{\pi} \right)^{2/3}.$$

Therefore, the Fermi energy of the neutrons in the neutron star of 2 solar masses and radius of 10 km will be

$$E_F = \frac{\left(6.63 \times 10^{-34}\right)^2}{8 \times 1.67 \times 10^{-27}} \times \left(\frac{3 \times 5.688 \times 10^{44}}{\pi} \right)^{2/3} \text{ J}$$

$$= 21.9 \times 10^{-12} \text{ J}$$

$$= 21.9 \times 10^{-12} \times 6.242 \times 10^{12} \text{ MeV}$$

$$= 136.7 \text{ MeV}.$$