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Problem 53.18 (RHK)

The Fermi energy of aluminium is 11.66 eV; *its density is* $2.70\ \text{g cm}^{-3}$ *and its molar mass is* $27.0\ \text{g mol}^{-1}$. *From these data, we have to determine the number of free electrons per atom.*

Solution:

We will calculate first the number of atoms of aluminium per cubic meter. It is given by the relation

$$n_{\text{Al}} = \frac{N_A \rho_{\text{Al}}}{M_{\text{Al}}} \ .$$

We use the following data:

$\rho_{\text{Al}} = 2.70 \times 10^3\ \text{kg m}^{-3}$,

and

$M_{\text{Al}} = 27.0 \times 10^{-3}\ \text{kg mol}^{-1}$.

We find

$$n_{\text{Al}} = \frac{N_A \rho_{\text{Al}}}{M_{\text{Al}}} = \frac{6.02 \times 10^{23} \times 2.70 \times 10^3}{27.0 \times 10^{-3}}\ \text{atoms per m}^3$$

$$= 6.02 \times 10^{28}\ \text{atoms per m}^3.$$

We calculate next the number density of conduction electrons per cubic meter from the Fermi energy E_F.

We recall that

$$E_F = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3},$$

where n is the number of conduction electrons per cubic meter. The Fermi energy of aluminium is 11.66 eV.

Therefore,

$$n = \frac{(8m)^{3/2}\,\pi E_F^{3/2}}{3h^3}$$

$$= \frac{\left(8\times 9.11\times 10^{-31}\right)^{3/2}\,\pi\left(11.66\times 1.6\times 10^{-19}\right)^{3/2}}{3\times\left(6.63\times 10^{-34}\right)^3} \text{ electrons per m}^3$$

$$= 1.80\times 10^{29} \text{ electrons per m}^3.$$

The number of conduction electrons per atom of aluminium will therefore be given by

$$\frac{n}{n_{\text{Al}}} = \frac{1.80\times 10^{29}}{6.02\times 10^{28}}$$

$$= 3.$$