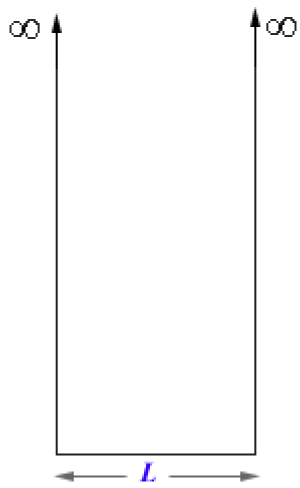
796.

Problem 53.11 (RHK)

Consider electrons trapped in an infinitely deep square well. Suppose that 100 *electrons are placed in a well of width* 120 pm, *two to a level with opposite spins. We have to calculate the Fermi energy of the system.* (*Note*: *The Fermi energy is the energy of the highest occupied level at the absolute zero of temperature*.)

Solution:

In an infinite potential well both ends $x = 0$ and $x = L$ are nodes of stationary waves. We therefore have the condition that



$$\frac{n\lambda}{2} = L,\ n = 1, 2, 3....$$

Therefore, the wavelengths of the stationary waves will be

$$\lambda_n = \frac{2L}{n},\ n = 1, 2, 3....$$

We using de Broglie relation we write the momentum associated with stationary waves,

$$p_n = \frac{h}{\lambda_n} = \frac{nh}{2L},\ n = 1, 2, 3....$$

The corresponding energy of a stationary state characterized with quantum number n is

$$E_n = \frac{p_n^2}{2m} = \frac{n^2 h^2}{8mL^2},\ n = 1, 2, 3...$$

As each energy state can be occupied by two electrons (spins in opposite directions), the quantum number of the highest occupied state in the well by 100 electrons will be $n = 50$.

Therefore, the Fermi energy of 100 electrons confined in an infinitely deep square well of width 120 pm will be

$$E_F = \frac{50^2 \times \left(6.63\times 10^{-34}\right)^2}{8\times 9.11\times 10^{-31}\times\left(120\times 10^{-12}\right)^2}\ \text{J}$$

$$= 1.047\times 10^{-14}\ \text{J} = 1.047\times 10^{-14}\times 6.242\times 10^{18}\ \text{eV}$$

$$= 6.53\times 10^{4}\ \text{eV}$$

$$= 65.3\ \text{keV}.$$

The Fermi energy E_F is 65.3 keV.