

794.

Problem 53.8 (RHK)

We have to prove that the occupancy probabilities for two states whose energies are equally spaced above and below the Fermi energy add up to one.

Solution:

The occupation probability in Fermi-Dirac statistics is

$$p = \frac{1}{\exp\left(\frac{E - E_F}{kT}\right) + 1},$$

where E_F is the Fermi energy.

Let $|E - E_F| = \Delta$. We have

$$\begin{aligned} p(E_F + \Delta) + p(E_F - \Delta) &= \frac{1}{\left(\exp(\Delta/kT) + 1\right)} + \frac{1}{\exp(-\Delta/kT) + 1} \\ &= \frac{2 + \exp(\Delta/kT) + \exp(-\Delta/kT)}{\left(\exp(\Delta/kT) + 1\right)\left(\exp(-\Delta/kT) + 1\right)} \\ &= 1. \end{aligned}$$