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Problem 53.7 (RHK)

The Fermi energy of silver is 5.5 eV. (a) *At* $T = 0^0\mathrm{C}$, *we have to calculate the probabilities that states at the following energies are occupied*: 4.4 eV, 5.4 eV, 5.5 eV, 5.6 eV, 6.4 eV. (b) *We have to find the temperature at which the probability that a state at* 5.6 eV *is occupied be* 0.16.

Solution:

For a gas of fermions the probability for occupation of a state of energy E at temperature T is given by the function

$$p = \frac{1}{\exp\left(\left(E - E_F\right)/kT\right) + 1},$$

where E_F is the Fermi energy.

At $0^0\mathrm{C}$ the temperature in Kelvin is 273 K. We calculate kT for $T = 273\ \mathrm{K}$. It is

$$kT = 8.62 \times 10^{-5} \times 273\ \mathrm{eV} = 0.0235\ \mathrm{eV}.$$

The Fermi energy of silver is 5.5 eV. We calculate the probability for occupation of states at the following energies: 4.4 eV, 5.4 eV, 5.5 eV, 5.6 eV, 6.4 eV.

$$p(4.4\text{ eV}) = \frac{1}{\exp(-1.1/0.0235)+1}$$

$$= \frac{1}{\exp(-46.8)+1} = 1.$$

$$p(5.4\text{ eV}) = \frac{1}{\exp(-0.1/0.0235)+1}$$

$$= \frac{1}{\exp(-4.255)+1} = \frac{1}{0.014+1} = 0.99.$$

$$p(5.5\text{ eV}) = \frac{1}{\exp(0)+1}$$

$$= \frac{1}{1+1} = 0.5\ .$$

$$p(5.6\text{ eV}) = \frac{1}{\exp(0.1/0.0235)+1}$$

$$= \frac{1}{\exp(4.255)+1} = \frac{1}{70.47+1} = 0.01\ .$$

$$p(6.4\text{ eV}) = \frac{1}{\exp(0.9/0.0235)+1}$$

$$= \frac{1}{\exp(38.29)+1} = 0\ .$$

(b)

We have to find the temperature at which the probability that a state at 5.6 eV is occupied will be 0.16. As

$$p = \frac{1}{\exp\left(\left(E - E_F\right)/kT\right) + 1},$$

we have

$$\exp\left(0.1\ \text{K}/8.62\times10^{-5}T\right) + 1 = \frac{1}{0.16},$$

or

$$\exp\left(1160\ \text{K}/T\right) = 5.25,$$

or

$$1160\ \text{K}/T = 1.658,$$

or

$$T = 699.6\ \text{K}$$