791.

Problem 53.5 (RHK)

The density and molar mass of sodium are $971\ \text{kg m}^{-3}$ *and* $23.0\ \text{g mol}^{-1}$, *respectively. We have to find* (a) *the fraction of the volume of the metallic sodium available to its conduction electrons*. (b) *We have to carry out the same calculation for copper. Its density, molar mass, and ionic radius are, respectively,* $8960\ \text{kg m}^{-3}$, $63.5\ \text{g mol}^{-1}$, *and* 96 pm. (c) *We have to answer in which of these two metals conduction electrons behave more like a free electron gas*.

Solution:

(a)

From the density and molar mass of sodium we calculate the number density of sodium atoms per unit volume. It is given by the equation

$$n_{\mathrm{Na}} = \frac{N_{\mathrm{A}}\rho_{\mathrm{Na}}}{\mathrm{M}_{\mathrm{Na}}}$$

$$= \frac{6.02\times10^{23}\ \mathrm{mol^{-1}}\times 971\ \mathrm{kg\ m^{-3}}}{23.0\times10^{-3}\ \mathrm{kg\ mol^{-1}}}$$

$$= 2.54\times10^{28}\,\mathrm{m^{-3}}.$$

As each atom of sodium contributes one conduction electron, n_{Na} is also the number density of conduction electrons per cubic meter in the sodium metal.

It is given that the radius of a $\mathrm{Na^{+}}$ ion is 98 pm. The total volume occupied by $\mathrm{Na^{+}}$ ions in a meter cube of metallic sodium will therefore be

$$V_{\mathrm{Na^{+}}} = \frac{4\pi}{3}\times n_{\mathrm{Na}}\times\left(r_{\mathrm{Na^{+}}}\right)^3$$

$$= \frac{4\pi}{3}\times 2.54\times10^{28}\times\left(98\times10^{-12}\right)^3\ \mathrm{m^3}$$

$$= 0.1\ \mathrm{m^3}.$$

Therefore, in 1 cubic mete of metallic sodium the volume available for conduction electrons will be $\left(1-0.1\right)\mathrm{m^3} = 0.9\ \mathrm{m^3}$, which is 0.9 times the total volume.

(b)

We do a similar calculation for copper.

$$n_{\mathrm{Cu}} = \frac{N_{\mathrm{A}}\rho_{\mathrm{Cu}}}{\mathrm{M}_{\mathrm{Cu}}}$$

$$= \frac{6.02\times10^{23}\ \mathrm{mol^{-1}}\times 8960\ \mathrm{kg\ m^{-3}}}{63.5\times10^{-3}\ \mathrm{kg\ mol^{-1}}}$$

$$= 8.49\times10^{28}\,\mathrm{m^{-3}}.$$

As each atom of copper contributes one conduction electron, n_{Cu} is also the number density of conduction electrons per cubic meter in the copper metal.

It is given that the radius of a Cu^{+} ion is 96 pm. The total volume occupied by Cu^{+} ions in a meter cube of metallic copper will therefore be

$$V_{\mathrm{Cu}^{+}} = \frac{4\pi}{3}\times n_{\mathrm{Cu}}\times\left(r_{\mathrm{Cu}^{+}}\right)^{3}$$

$$= \frac{4\pi}{3}\times 8.49\times10^{28}\times\left(96\times10^{-12}\right)^{3}\ \mathrm{m^{3}}$$

$$= 0.31\ \mathrm{m^{3}}.$$

Therefore, the fraction of the volume of metallic copper available to its conduction electrons is $(1-0.31)=0.69$.

(c)

From these results we conclude that in metallic sodium conduction electrons will behave more like a free electron gas than the conduction electrons of metallic copper.