788.

Problem 53.1 (RHK)

We have to show (a) *that the function* $n(E)$ *for the density of states for a quantum free electron gas*

$$n(E)=\frac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2}$$

can be written as

$$n(E)=CE^{1/2},$$

where $C=6.80\times10^{27}\ \mathrm{m^{-3}\ eV^{-3/2}}$.

(b) *Using this relation we have to show that for* $E=5.00\ \mathrm{eV}$, $n(E)=1.52\times10^{28}\ \mathrm{m^{-3}\ eV^{-1}}$.

Solution:

(a)

In the formula for $n(E)$ we will substitute the values of the fundamental constants, mass of electron, $m=9.11\times10^{-31}\ \mathrm{kg}$, Planck constant, $h=6.63\times10^{-34}\ \mathrm{J\ s}$. We find

$$C = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} = \frac{8\sqrt{2}\pi \times \left(9.11\times 10^{-31}\ \text{kg}\right)^{3/2}}{\left(6.63\times 10^{-34}\ \text{J s}\right)^3}$$

$$= 3.353\times 10^{55.5}\left(\text{kg}^{3/2} / \text{kg}^3\ \text{m}^6\ \text{s}^{-3}\right)$$

$$= 3.353\times 10^{55.5}\ \text{m}^{-3}\ .\ \left(\text{kg m}^2\ \text{s}^{-2}\right)^{-3/2}$$

$$= 3.353\times 10^{55.5}\ \text{m}^{-3}.\text{J}^{-3/2}$$

$$= 3.353\times 10^{55.5}\text{m}^{-3}\times\left(6.242\times 10^{18}\ \text{eV}\right)^{-3/2}$$

$$= 2.15\times 10^{-1}\times 10^{28.5}\text{m}^{-3}\ \text{eV}^{-3/2}$$

$$= 6.80\times 10^{27}\text{m}^{-3}\ \text{eV}^{-3/2}.$$

(b)

Substituting $E = 5.00\ \text{eV}$ in the above formula for $n(E)$, using $C = 6.80\times 10^{27}\ \text{m}^{-3}\ \text{eV}^{-3/2}$, we find

$$n(E) = 6.80\times 10^{27}\times 5^{1/2}\ \text{m}^{-3}\ \text{eV}^{-1} = 1.52\times 10^{28}\ \text{m}^{-3}\ \text{eV}^{-1}.$$