786.

<u>Problem 52.34 (RHK)</u>

The beam from an argon laser $(\lambda = 515\ \text{nm})$ *has a diameter* d *of* 3.00 mm *and a power output of* 5.21 W. *The beam is focussed onto a diffuse surface by a lens of focal length* $f = 3.50\ \text{cm}$. *A diffraction pattern is formed*. (a) *We have to show that the radius of the central disk is given by*

$$R = \frac{1.22 f \lambda}{d}.$$

The central disk can be shown to contain 84% *of the incident power*. *We have to calculate* (b) *the radius of the central disk, and the average power flux density* (c) *in the incident beam and* (d) *in the central disk.*

<u>Solution:</u>

(a)

A result of Fraunhofer diffraction is that image formed of a distant object by a lens at its focal point is not a point but a circular disk surrounded by several progressively fainter secondary rings. The first minimum occurs at an angle from the central axis given by

$$\sin\theta = 1.22\frac{\lambda}{d},$$

where d is the diameter of the aperture. As $\lambda/d =$ 1,

$\sin\theta$; $\theta = 1.22\frac{\lambda}{d}$. As the diffraction ring is formed at the focal plane of the lens, its radius will be

$$R = f\theta = \frac{1.22 f\lambda}{d} \; .$$

(b)

We calculate the radius of the central disk using the data of the problem:

$f = 3.50$ cm,

$\lambda = 515$ nm,

and

$d = 3.00$ mm.

We find

$$R = \frac{1.22\times 3.50\times 10^{-2}\times 515\times 10^{-9}}{3.00\times 10^{-3}} \text{ m}$$

$$= 733\times 10^{-8} \text{ m} = 7.33\ \mu\text{m}.$$

(c)

The average power flux density in the incident beam,

$$\frac{\text{power output of the laser}}{\text{cross-sectional area of the beam}} = \frac{5.00}{(\pi/4)\left(3.00\times10^{-3}\right)^2}\ \text{W m}^{-2}$$

$$= 7.07\times10^{5}\ \text{W m}^{-2}.$$

(d)

It is given that the central disk receives about 84% of the incident power of the beam. The power of the central disk will therefore be $5.0\times0.84\ \text{W} = 4.2\ \text{W}$.

Therefore, the average power flux density in the central disk will be

$$= \frac{\text{average power received in the central disk}}{\text{area of the central disk}}$$

$$= \frac{4.2}{\pi(R)^2}\ \text{W m}^{-2} = \frac{4.2}{\pi\times\left(7.33\times10^{-6}\right)^2}\ \text{W m}^{-2}$$

$$= 2.49\times10^{10}\ \text{W m}^{-2}.$$