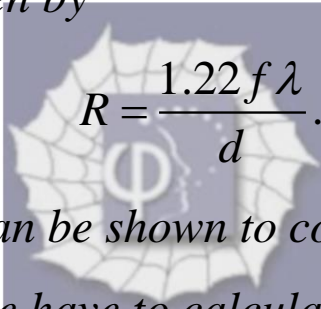


786.

**Problem 52.34 (RHK)**

The beam from an argon laser ( $\lambda = 515 \text{ nm}$ ) has a diameter  $d$  of 3.00 mm and a power output of 5.21 W. The beam is focussed onto a diffuse surface by a lens of focal length  $f = 3.50 \text{ cm}$ . A diffraction pattern is formed. (a) We have to show that the radius of the central disk is given by


$$R = \frac{1.22 f \lambda}{d}.$$

The central disk can be shown to contain 84% of the incident power. We have to calculate (b) the radius of the central disk, and the average power flux density (c) in the incident beam and (d) in the central disk.

**Solution:**

(a)

A result of Fraunhofer diffraction is that image formed of a distant object by a lens at its focal point is not a point but a circular disk surrounded by several progressively

fainter secondary rings. The first minimum occurs at an angle from the central axis given by

$$\sin \theta = 1.22 \frac{\lambda}{d},$$

where  $d$  is the diameter of the aperture. As  $\lambda/d = 1$ ,

$\sin \theta$ ;  $\theta = 1.22 \frac{\lambda}{d}$ . As the diffraction ring is formed at the

focal plane of the lens, its radius will be

$$R = f\theta = \frac{1.22 f \lambda}{d}.$$

(b)

We calculate the radius of the central disk using the data of the problem:

$$f = 3.50 \text{ cm},$$

$$\lambda = 515 \text{ nm},$$

and

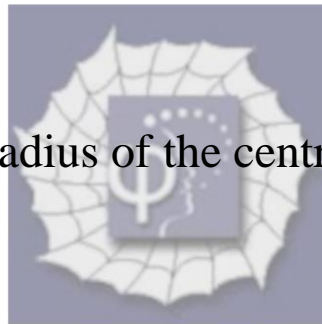
$$d = 3.00 \text{ mm}.$$

We find

$$\begin{aligned} R &= \frac{1.22 \times 3.50 \times 10^{-2} \times 515 \times 10^{-9}}{3.00 \times 10^{-3}} \text{ m} \\ &= 733 \times 10^{-8} \text{ m} = 7.33 \mu\text{m}. \end{aligned}$$

(c)

The average power flux density in the incident beam,



$$\frac{\text{power output of the laser}}{\text{cross-sectional area of the beam}} = \frac{5.00}{(\pi/4)(3.00 \times 10^{-3})^2} \text{ W m}^{-2}$$

$$= 7.07 \times 10^5 \text{ W m}^{-2}.$$

(d)

It is given that the central disk receives about 84% of the incident power of the beam. The power of the central disk will therefore be  $5.0 \times 0.84 \text{ W} = 4.2 \text{ W}$ .

Therefore, the average power flux density in the central disk will be

$$= \frac{\text{average power received in the central disk}}{\text{area of the central disk}}$$

$$= \frac{4.2}{\pi(R)^2} \text{ W m}^{-2} = \frac{4.2}{\pi \times (7.33 \times 10^{-6})^2} \text{ W m}^{-2}$$

$$= 2.49 \times 10^{10} \text{ W m}^{-2}.$$