780.

Problem 52.30 (RHK)

A population inversion for two levels is often described by assigning a negative Kelvin temperature to the system. We have to show that such a negative temperature would indeed correspond to an inversion. A three-level laser emits laser light at a wavelength of 550 nm. *We have to find the negative temperature that would describe the system if the population of the upper level exceeds that of the lower by* 10.0%.

Solution:

The energy difference between the upper and the lower levels is determined by the energy of the photon, which corresponds to a wavelength of 550 nm. We have

$$E_2 - E_1 = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{550\times10^{-9}}\ \text{J} = 3.62\times10^{-19}\ \text{J}\ .$$

Let the population of the lower level be $n(E_1)$. As the population of the upper level $n(E_2)$ exceeds that of the lower level by 10%, we have

$$\frac{n(E_2)}{n(E_1)} = 1.10 \ .$$

Let the system be described by temperature $-|T|$ K.

From the Boltzmann distribution, we have

$$\frac{n(E_2)}{n(E_1)} = \exp\left(-(E_2 - E_1)/kT\right) = 1.10,$$

or

$$\exp\left(3.62\times10^{-19}/1.38\times10^{-23}|T|\right) = 1.10,$$

or

$$T = -\frac{2.62\times10^{4}}{\ln 1.10} \text{ K} = -\frac{2.62\times10^{4}}{9.53\times10^{-2}} \text{ K} = -2.75\times10^{5} \text{ K}.$$