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Problem 52.14 (RHK)

In an x-ray tube an electron moving initially at a speed of $2.73\times10^{8}\ \mathrm{m\,s^{-1}}$ *slows down in passing near a nucleus. A single photon of energy* 43.8 keV is *emitted. We have to find the final speed of the electron.* (*Relativity must be taken into account*; *energy imparted to the nucleus may be ignored*.)

Solution:

The relativistic energy of an electron moving with speed v is given by the relation

$$E=\frac{m_e c^2}{\sqrt{1-\left(v/c\right)^2}}\ .$$

The energy of the electron when it is moving with speed of $2.73\times10^{8}\ \mathrm{m\,s^{-1}}$ will be

$$E_i=\frac{0.511}{\sqrt{1-\left(2.73\times10^{8}/3\times10^{8}\right)^2}}\ \mathrm{MeV}$$

$$=1232.5\ \mathrm{keV}.$$

The energy of the electron after it has emitted a photon of energy 43.8 keV will be (neglecting energy imparted to the nucleus)

$$E_f = (1232.5 - 43.8)\,\text{keV} = 1{,}188.7\ \text{keV}\ .$$

The speed of the electron after it has emitted the electron will therefore be

$$1-(v_f/c)^2 = \left(\frac{m_e c^2}{E_f}\right)^2$$

$$= \left(\frac{511}{1188.7}\right)^2$$

$$= 0.1848.$$

Therefore,

$$v_f = 2.71\times 10^8\ \text{m s}^{-1}\ .$$