770.

<u>Problem 51.59 (RHK)</u>

For the state $n = 2,\ l = 0$, (a) *we have to locate the two maxima for the radial probability density curve*; *and* (b) *calculate the radial probability density at the two maxima*.

<u>Solution:</u>

(a)

The wave function for the state $n = 2$ and $l = 0$ is

$$\psi_{200}(r) = \frac{1}{4\sqrt{2\pi a^3}}(2 - r/a)e^{-r/2a}\ .$$

The radial probability density $P_r(r)$ is given by

$$P_r(r) = \psi_{200}^2(r)\left(4\pi r^2\right)$$

$$= \frac{1}{8a}\left(\frac{r}{a}\right)^2 (2 - r/a)^2\, e^{-r/a}.$$

For finding out the maxima of $P_r(r)$, we calculate

$$\frac{dP_r(r)}{dr}.$$

We find

$$\frac{dP_r(r)}{dr} = \frac{1}{8a^2}\left(\frac{r}{a}\right)\left(2-\frac{r}{a}\right)e^{-r/a}\left\{4-\frac{6r}{a}+\left(\frac{r}{a}\right)^2\right\}.$$

It can be shown that $r = 2a$ is a minimum and the maxima of $P_r(r)$ are the roots of the equation

$$4-\frac{6r}{a}+\left(\frac{r}{a}\right)^2 = 0,$$

which are

$$\frac{r}{a} = 3-\sqrt{5},\ 3+\sqrt{5}$$

$$= 0.76,\ 5.24\ .$$

(b)

The radial probability density at the first maxima $r = 0.76a$ will be

$$P_r(r = 0.76a) = \frac{1}{8a}(0.76)^2(2-0.76)^2 e^{-0.76}$$

$$= 0.98\ (\mathrm{nm})^{-1},$$

where we have used

Bohr radius $a = 5.292\times10^{-2}$ nm.

The radial probability density at the first maxima $r = 5.24a$ will be

$$P_r(r = 5.24a) = \frac{1}{8a}(5.24)^2(2-5.24)^2 e^{-5.24}$$

$$= 3.61\ (\mathrm{nm})^{-1}.$$