

769.

Problem 51.66 (RHK)

We will apply Bohr's model to the positronium atom. This consists of a positive and a negative electron revolving around the centre of mass, which lies halfway between them. (a) We have to find the relation between this spectrum and the hydrogen spectrum. (b) We have to find the radius of the ground-state orbit.

Solution:



This problem can be answered by replacing the electron mass m_e in the results of the Bohr's model of the hydrogen atom by the reduced mass

$$\mu = \frac{m_e m_e}{m_e + m_e} = \frac{m_e}{2}.$$

The energies of the positronium states using the Bohr's model will therefore be given by

$$\begin{aligned}
E_{\text{positronium}}(n) &= -\frac{1}{2} \frac{\mu c^2}{n^2} \alpha^2 = -\frac{1}{4} \frac{m_e c^2}{n^2} \alpha^2 \\
&= \frac{1}{2} \left(-\frac{1}{2} \frac{m_e c^2}{n^2} \alpha^2 \right) \\
&= -\frac{13.6}{2n^2} \text{ eV} \\
&= \frac{E_{\text{hydrogen}}(n)}{2}.
\end{aligned}$$

(b)

As the mass of the electron m_e in the hydrogen atom relations is replaced by the reduced by $\mu = m_e/2$ in the positronium, the radius of the ground-state orbit of the positronium will be twice that of the hydrogen atom and therefore will be 1.058×10^{-10} m.