765.

Problem 51.65 (RHK)

By applying Bohr's model to a muonic atom, which consists of a nucleus of charge Ze *with a negative muon* (*an elementary particle with a charge* $q = -e$ *and a mass* $m = 207m_e$, *where* m_e *is the electron mass*) *circulating about it, we have to calculate* (a) *the muon-nucleus separation in the first Bohr orbit*, (b) *the ionization energy, and* (c) *the wavelength of the most energetic photon that can be emitted. We may assume that the muon is circulating about a hydrogen nucleus* $(Z = 1)$.

Solution:

We will take into account the finite mass of the nucleus by using the reduce mass for the mass of the orbiting muon, that is

$$m = \frac{207m_e}{1 + \frac{207}{1836}} = \frac{207m_e}{1 + 0.113} = 186m_e.$$

(a)

The muon-nucleus separation in the first Bohr orbit will be given by

$$a_{muon} = \frac{a}{186} = \frac{0.529\times10^{-11}}{186}\ \text{m} = 2.84\times10^{-14}\ \text{m}$$

$$= 28.4\ \text{fm.}$$

We have used that the Bohr radius

$$a = \frac{\mathrm{h}^2}{m_e\left(e^2/4\pi\varepsilon_0\right)} = 0.529\times10^{-11}\ \text{m.}$$

(b)

The ionization energy of the muonic atom will be

$$E_{ion(muon)} = \frac{1}{2}(186m_e)c^2\alpha^2 = 13.6\times186\ \text{eV}$$

$$= 2.53\ \text{kV.}$$

(c)

The wavelength of the most energetic photon that can be emitted by a muonic atom will be

$$\lambda = \frac{c}{E_{ion(muon)}/h} = \frac{hc}{E_{ion(muon)}} = \frac{6.63\times10^{-34}\times3\times10^8}{2.53\times10^3\times1.6\times10^{-19}}\ \text{m}$$

$$= 4.91\times10^{-10}\ \text{m}$$

$$= 0.491\ \text{nm.}$$