764.

Problem 40.51P (HRW)

The wave functions for the three states of hydrogen atom, which have $n=2,\ l=1,\ m_l=0,\ +1,$ *and* -1, *are*

$$\psi_{210}(r,\theta)=\left(1/4\sqrt{2\pi}\right)\left(a^{-3/2}\right)(r/a)e^{-r/2a}\cos\theta,$$

$$\psi_{21+1}(r,\theta,\phi)=\left(1/8\sqrt{\pi}\right)\left(a^{-3/2}\right)(r/a)e^{-r/2a}\sin\theta e^{i\phi},$$

$$\psi_{21-1}(r,\theta,\phi)=\left(1/8\sqrt{\pi}\right)\left(a^{-3/2}\right)(r/a)e^{-r/2a}\sin\theta e^{-i\phi},$$

in which the subscripts on $\psi(r,\theta,\phi)$ *give the values of the quantum numbers* $n,\ l,\ m_l$. *Note that the first wave function is real but the others, which involve the imaginary number* i, *are complex.*

We have to find the probability density for each function and by adding the three probability densities show that their sum is spherically symmetric, depending on the radial coordinate r.

Solution:

(a)

The probability density for a quantum state is given by the modulus square of the normalized wave function.

The probability densities of the states corresponding to the quantum numbers, $n=2$, $l=1$, $m_l=0$, +1, and -1, will therefore be

$$P_{210}(r,\theta,\phi)=\left|\psi_{210}(r,\theta,\phi)\right|^2=\frac{1}{32\pi}\frac{1}{a^5}r^2e^{-r/a}\cos^2\theta,$$

$$P_{211}(r,\theta,\phi)=\left|\psi_{211}(r,\theta,\phi)\right|^2=\frac{1}{64\pi}\frac{1}{a^5}r^2e^{-r/a}\sin^2\theta,$$

and

$$P_{21-1}(r,\theta,\phi)=\left|\psi_{21-1}(r,\theta,\phi)\right|^2=\frac{1}{64\pi}\frac{1}{a^5}r^2e^{-r/a}\sin^2\theta.$$

Therefore, we get

$$\left|\psi_{210}(r,\theta,\phi)\right|^2+\left|\psi_{211}(r,\theta,\phi)\right|^2+\left|\psi_{21-1}(r,\theta,\phi)\right|^2$$

$$=\frac{1}{32\pi}\frac{1}{a^5}r^2e^{-r/a}\cos^2\theta+\frac{1}{32\pi}\frac{1}{a^5}r^2e^{-r/a}\sin^2\theta$$

$$=\frac{1}{32\pi}\frac{1}{a^5}r^2e^{-r/a}.$$