763.

<u>Problem 51.56 (RHK)</u>

In the ground state of the hydrogen atom we have to show that the probability P *that the electron lies within a sphere of radius r is given by*

$$\mathrm{P} = 1 - e^{-2x}\left(1 + 2x + 2x^2\right),$$

in which $x = r/a$. (b) *We have to evaluate the probability that, in the ground state, the electron lies within a sphere of radius a.*

<u>Solution:</u>

(a)

The radial probability density $P(r)$ in the ground state of the hydrogen atom is given by the expression

$$P(r) = \frac{4r^2}{a^3} e^{-2r/a} \ ,$$

in which a is the Bohr radius.

Therefore, the probability that the electron lies within a sphere of radius *r* will be given by the integral

$$\mathrm{P} = \int_0^r P(r)dr .$$

We thus have

$$\mathrm{P} = \int_0^r P(r)dr = \frac{4}{a^3}\int_0^r r^2 e^{-2r/a} dr$$

$$= \frac{4}{a^3}\left\{ \left(\frac{r^2 e^{-2r/a}}{(-2/a)} \right]_0^r - \int_0^r \frac{2re^{-2r/a}}{(-2/a)} dr \right\}$$

$$= \frac{4}{a^3}\left\{ \begin{array}{l} \left(\frac{-a}{2}\right)\left(r^2 e^{-2r/a}\right) + a\left(\frac{re^{-2r/a}}{(-2/a)} \right]_0^r \\ -a\int_0^r \frac{e^{-2r/a}}{(-2/a)} dr \end{array} \right\}$$

$$= \left\{ \begin{array}{l} -2\left(\frac{r}{a}\right)^2 e^{-2r/a} - 2\left(\frac{r}{a}\right) e^{-2r/a} \\ -\left(e^{-2r/a} - 1\right) \end{array} \right\}$$

$$= 1 - e^{-2r/a}\left(1 + 2\left(\frac{r}{a}\right) + 2\left(\frac{r}{a}\right)^2 \right).$$

Therefore,

$$\mathrm{P} = 1 - e^{-2x}\left(1 + 2x + 2x^2\right), \text{ where } x = r/a .$$

(b)

The probability that the electron lies within a sphere of Bohr radius *a* will therefore be given by

$x = 1.$

$$P(a) = 1 - e^{-2}(1 + 2 + 2)$$

$$= 1 - 5e^{-2} = 0.323.$$

The probability that electron will be found within a sphere of one Bohr radius is 32.3%.