762.

Problem 51.54 (RHK)

In atoms there is a finite, though very small, probability that, at some instant, an orbital electron will actually be found inside the nucleus. In fact, some unstable nuclei use this occasional appearance of the electron to decay by electron capture. Assuming that the proton itself is a sphere of radius 1.1×10^{-15} m *and that the hydrogen atom electron wave function holds all the way to the proton's centre, using the ground state wave function we have to calculate the probability that the hydrogen atom electron is inside the nucleus.*

Solution:

The radial probability density for electron in the ground state of the hydrogen atom is given by the function

$$P(r) = \frac{4r^2 e^{-2r/a}}{a^3},$$

where a is the Bohr radius.

$$a = \frac{\mathrm{h}^2}{m\left(e^2/4\pi\varepsilon_0\right)} = 5.292\times10^{-11} \text{ m}.$$

Within the sphere of radius 1.1×10^{-15} m, as

$$\frac{1.1\times10^{-15}}{5.292\times10^{-11}}=2.0\times10^{-5},$$

We can approximate the radial probability density as

$$P(r)\approx\frac{4r^2}{a^3},\ \text{for } e^{-2.0\times10^{-5}}\ ;\ 1.$$

Therefore, the probability of finding the electron inside the proton, in the ground state of the hydrogen atom, will be

$$\text{P}=\frac{4}{a^3}\int_0^{1.1\times10^{-15}} r^2dr=\frac{4}{3}\times\left(\frac{1.1\times10^{-15}}{5.3\times10^{-11}}\right)^3$$

$$=\frac{4}{3}\times\left(2.0\times10^{-5}\right)^3$$

$$=10.6\times10^{-15}.$$