760.

Problem 40.49P (HRW)

We have to show that the average value of r, defined as

$$\bar{r} = \int_0^\infty rP(r)dr,$$

has the value 1.5*a*. $P(r)$ *is the radial probability density for the ground state of the hydrogen atom*.

Solution:

The radial probability density for the ground state of the hydrogen atom is

$$P(r) = \frac{4}{a^3}e^{-2r/a},$$

where *a* is the Bohr radius.

Therefore,

$$\bar{r} = \int_0^\infty rP(r)dr = \frac{4}{a^3}\int_0^\infty r^3 e^{-2r/a}dr.$$

We evaluate the integral by making the following substitutions:

$$\frac{2r}{a} = \xi,\ dr = \frac{a}{2} d\xi.$$

We get

$$\bar{r} = \frac{4}{a^3}\left(\frac{a}{2}\right)^4 \int_0^\infty \xi^3 e^{-\xi} d\xi = \frac{a}{4}\Gamma(4) = \frac{a}{4} 3! = \frac{6a}{4} = 1.5a.$$