759.

<u>Problem 40.43P (HRW)</u>

We have to calculate the probability that the electron in the hydrogen atom, in its ground state, will be found between spherical shells whose radii are a and 2*a, where a is the Bohr radius*.

<u>Solution:</u>

The radial probability density $P(r)$ for a hydrogen atom is defined such that $P(r)dr$ gives the probability for the electron to be found in the volume enclosed by concentric spherical shell of radius *r* and thickness dr. For the hydrogen atom in the ground state, we have

$$P(r) = \frac{4}{a^3} r^2 e^{-2r/a},$$

in which a is the Bohr radius. We recall that

$$a = \frac{\mathrm{h}^2}{m\left(e^2/4\pi\varepsilon_0\right)} = 52.9 \text{ pm}.$$

Therefore, the probability of finding electron in the volume between the spherical shells of radii *a* and 2*a* will be given by the following integral:

$$P_{a\le r\le 2a} = \int_a^{2a} P(r)\,dr = \frac{4}{a^3}\int_a^{2a} r^2 e^{-2r/a}\,dr$$

$$= \frac{4}{a^3}\left\{\left(\left.\frac{r^2 e^{-2r/a}}{(-2/a)}\right]_a^{2a} - \int_a^{2a}\frac{2re^{-2r/a}}{(-2/a)}\,dr\right\}$$

$$= \frac{4}{a^3}\left\{\begin{array}{l} -\frac{a}{2}\left(4a^2e^{-4} - a^2e^{-2}\right) + a\left(\left(\left.\frac{re^{-2r/a}}{(-2/a)}\right]_a^{2a}\right)\right) \\ -a\int_a^{2a}\frac{e^{-2r/a}dr}{(-2/a)} \end{array}\right\}$$

$$= \frac{4}{a^3}\left\{\begin{array}{l} -\frac{a^3}{2}\left(4e^{-4} - e^{-2}\right) - \frac{a^2}{2}\left(2ae^{-4} - ae^{-2}\right) + \\ \frac{a^2}{2}\left(\left.\frac{e^{-2r/a}}{(-2/a)}\right]_a^{2a} \end{array}\right\}$$

$$= \left\{-2\left(4e^{-4} - e^{-2}\right) - 2\left(2e^{-4} - e^{-2}\right) - \left(e^{-4} - e^{-2}\right)\right\}$$

$$= \left\{-13e^{-4} + 5e^{-2}\right\} = 0.4386.$$

The probability of finding the electron in the hydrogen atom, in its ground state, between spherical shells whose radii are *a* and 2*a* will, therefore, be 43.9%.