758.

Problem 40.42P (HRW)

We have to calculate the probability that in the ground state of the hydrogen atom, the electron will be found at a radius greater than the Bohr radius.

Solution:

The normalized wave function for the ground state of the hydrogen atom is

$$\psi(r) = \frac{1}{\sqrt{\pi}a^{3/2}} e^{-r/a}.$$

Here, a is the Bohr radius and is given in terms of fundamental constants by the expression

$$a = \frac{\mathrm{h}^2}{m\left(e^2/4\pi\varepsilon_0\right)} = 5.29\times10^{-11}\ \text{m}.$$

The radial probability density

$$P(r) = \psi^2(r)\times\left(4\pi r^2\right)$$

$$= \frac{4r^2}{a^3} e^{-2r/a}.$$

Therefore, the probability for finding the electron at a radius greater than a in the hydrogen atom will be

$$P_{r>a} = \int_a^{\infty} \frac{4r^2}{a^3} e^{-2r/a} dr$$

$$= \frac{4}{a^3}\left(\left(-\frac{r^2 e^{-2r/a}}{2/a}\right]_a^{\infty} + \int_a^{\infty} are^{-2r/a} dr\right)$$

$$= \frac{4}{a^3}\left(\frac{a^3 e^{-2}}{2} + a\left(-\frac{re^{-2r/a}}{2/a}\right]_a^{\infty} + \frac{a^2}{2}\int_a^{\infty} e^{-2r/a} dr\right)$$

$$= \frac{4}{a^3}\left(\frac{a^3 e^{-2}}{2} + \frac{a^3 e^{-2}}{2} + \frac{a^2}{2}\left(-\frac{e^{-2r/a}}{2/a}\right]_a^{\infty}\right)$$

$$= \frac{4}{a^3}\left(a^3 e^{-2} + \frac{a^3}{4} e^{-2}\right) = 5e^{-2} = 0.67.$$