757.

Problem 40.19P (HRW)

The probability density for the region $x > L$ *of a square well of width L,* $U = 0$ $, 0 < x < L$, *and* $U = E_{\text{pot}}$, $x < 0$, $x > L$ *, drops off exponentially according to*

$$\psi^2(x) = Ce^{-2kx},$$

where C is a constant. (a) *We have to show that the wave function* $\psi(x)$ *is a solution of the one-dimensional form of the Schrödinger's equation*, *and have to find the value of k for this to be true.*

Solution:

The one-dimensional Schrödinger equation for a particle of mass *m*, energy *E*, moving in the square well potential $U = 0$ $, 0 < x < L$, $U = E_{\text{pot}}$, $x < 0$, $x > L$, in the region $x > L$ will be

$$\frac{d^2\psi}{dx^2} - \frac{8\pi^2 m}{h^2}(E_{\text{pot}} - E)\psi = 0.$$

If $\psi^2(x) = Ce^{-2kx}$, we have $\psi(x) = C'e^{-kx}$.

Substituting $\psi(x) = C'e^{-kx}$ in the Schrödinger equation given above, we get

$$C'\left(k^2 e^{-kx} - \frac{8\pi^2 m}{h^2}\left(E_{\text{pot}} - E\right)e^{-kx}\right) = 0.$$

This implies that

$$\left(k^2 - \frac{8\pi^2 m}{h^2}\left(E_{\text{pot}} - E\right)\right) = 0.$$

or

$$k^2 = \frac{8\pi^2 m}{h^2}\left(E_{\text{pot}} - E\right),$$

and

$$k = \sqrt{\frac{8\pi^2 m}{h^2}\left(E_{\text{pot}} - E\right)}\ .$$