756.

Problem 51.47 (RHK)

The proton as well as the electron has spin $\frac{1}{2}$. *In the hydrogen atom in its ground state, with* $n=1$ *and* $l=0$, *there are two energy levels, depending on whether the electron and the proton spins are in the same direction or opposite directions. The state with the spins in the opposite direction has the higher energy. If an atom is in this state and one of the spins* "*flips over*", *the small energy difference is released as a photon of wavelength* 21 cm. *This spontaneous spin-flip process is very slow, the mean life for the process being* 10^7 y. However, *radio astronomers observe this* 21-cm *radiation from interstellar space, where the density of hydrogen is so small that an atom can flip before being disturbed by the collisions with the other atoms. We have to calculate the effective magnetic field* (*due to the magnetic dipole moment of the proton*) *experienced by the electron in the emission of this* 21-cm *radiation.*

Solution:

The magnetic moment associated with spin angular momentum of an electron is

$$\vec{\mu}_s = -\frac{e}{m_e}\vec{S}\,.$$

If we quantize the spin in the direction of the magnetic field of the proton, the energy values of the two spin states are given by

$$\vec{\mu}_s.\vec{B} = -\frac{ehB}{4\pi m_e},\ \ \frac{ehB}{4\pi m_e}\,.$$

Therefore, the energy of the photon emitted when there is a spin-flip of the electron will be

$$h\nu = 2\times\frac{ehB}{4\pi m_e}.$$

We express the magnitude of the magnetic field experienced by the electron in terms of the wavelength of the emitted radiation. We have the relation

$$B = \frac{hc}{2\lambda\mu_{\mathrm{B}}},$$

where μ_{B} is the Bohr magneton and its value is

$$\mu_{\mathrm{B}} = \frac{eh}{4\pi m_e} = 9.274\times10^{-24}\ \mathrm{J\ T^{-1}}\ .$$

As $\lambda = 21\ \mathrm{cm} = 21\times10^{-2}\ \mathrm{m}$, we find

$$B = \frac{6.6\times10^{-34}\times3\times10^{8}}{2\times21\times10^{-2}\times9.274\times10^{-24}}\ \text{T} = 5.08\times10^{-2}\ \text{T}.$$