752.

Problem 51.22 (RHK)

In stars the Pickering series is found in the He^+ *spectrum. It is emitted when the electron in* He^+ *jumps from higher levels to the level with* $n = 4$. *We have to show that* (a) *the wavelengths of the lines in this series are given by*

$$\lambda = \frac{1}{R} \times \frac{4n^2}{n^2 - 16},$$

in which $n = 5, 6, 7\ldots$ (b) *We have to calculate the wavelength of the first line in this series and of the series limit.* (c) *We have to find the region of the electromagnetic spectrum in which this series occurs.*

Solution:

(a)

In the Bohr' model the energies of hydrogen like atom are given by

$$E_n = -\frac{hcR}{n^2}, \ n = 1, 2, 3\ldots$$

The Rydberg constant R can be expressed in terms of the fundamental constants as

$$R_{\mathrm{He}^+} = \frac{2\alpha^2}{(h/mc)} = 4R$$

where Z is the charge of the nucleus and α is the fine structure constant,

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \mathrm{h}c} = \frac{1}{137}.$$

For He^+ atom $Z = 2$ and

$$R = \frac{2\alpha^2}{(h/mc)}.$$

Pickering series corresponds to transitions from states with $n = 5, 6, 7..$ to the state with $n = 4$. Thus the wavelengths of radiation which form Pickering series will be given by

$$\lambda_n = \frac{c}{\nu_n} = \frac{ch}{\left(\frac{hcR_{\mathrm{He}^+}}{4^2} - \frac{hcR_{\mathrm{He}^+}}{n^2}\right)} = \frac{4n^2}{R\left(n^2 - 16\right)}.$$

(b)

The wavelength of the first line in the Pickering series will correspond to transition from the state with $n = 5$ to $n = 4$. We find

$$\lambda_5 = \frac{4\times 25}{R(25-16)} = \frac{100}{9R}.$$

We next calculate the value of R for

Therefore,

$$\lambda_5 = \frac{100}{9} \times \frac{2\times 2.418\times 10^{-12}}{(1/137)^2}\,\mathbf{m} = 1008\ \ \mathbf{nm.}$$

Wavelength of the series limit will correspond to $n \to \infty$.

We get

$$\lambda_{\text{series limit}} = \frac{4}{R} = 2\times 4\times (137)^2 \times 2.418\times 10^{-12}\ \ \mathbf{m}$$
$$= 36.28\ \ \mathbf{nm.}$$

(c)

The spectrum of wavelengths in the Pickering series, therefore, occurs in the ultraviolet region extending to visible region.