750.

<u>Problem 51.16 (RHK)</u>

We have to calculate, according to the Bohr model, (a) *the speed of the electron in the ground state of the hydrogen atom*; (b) *the corresponding de Broglie wavelength*; *and* (c) *find a relation between the de Broglie wavelength* λ *and the radius* a_0 *of the ground-state Bohr orbit*.

<u>Solution:</u>

We will first setup the Bohr model of atom. In the Bohr model of atom we consider an electron moving in circular orbit about the positively charged proton. According to the Newton's second law, we have

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2},$$

where v is the speed of the electron in the orbit of radius r. We restrict the orbits by the Bohr's quantization condition of angular momentum. The Bohr quantization condition is

$$l = mvr = n\text{h},\ n = 1,2,3...$$

From the above two equations, we find

$$v = \frac{c}{n} \times \alpha,$$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \mathrm{h} c} = \frac{1}{137},$$

and

$$r = n^2 \times \left(\frac{4\pi\varepsilon_0 \mathrm{h}^2}{me^2} \right).$$

The ground-state of the hydrogen atom corresponds to $n = 1$.

(a)

Therefore, the speed of the electron in the ground state of the hydrogen atom, in the Bohr' model, is

$$v_1 = \alpha c = \frac{1}{137} c.$$

And the radius of the ground-state orbit is

$$r_1 = a_0 = \frac{\mathrm{h}^2}{me^2/4\pi\varepsilon_0}.$$

(b) and (c)

The de Broglie wavelength of the electron in the ground-state will be

$$\lambda = \frac{h}{mv_1} = \frac{h}{mc\alpha} = \frac{h}{mc} \times \left(\frac{(4\pi\varepsilon_0)\mathrm{h}c}{e^2} \right)$$

$$= \frac{2\pi\mathrm{h}^2 \times (4\pi\varepsilon_0)}{me^2} = 2\pi a_0,$$

or

$$\lambda = 2\pi a_0 \ .$$

We find that the de Broglie wavelength of the electron in the ground-state of the hydrogen atom in the Bohr' model is equal to the circumference of the orbit of radius a_0.