748.

Problem 51.13 (RHK)

If an electron is revolving in an orbit at frequency ν_0, classical electromagnetism predicts that it will radiate energy not only at this frequency but also at $2\nu_0$, $3\nu_0$, $4\nu_0$, and so on. We have to show that this is also predicted by Bohr's theory of the hydrogen atom in the limiting case of large quantum numbers.

Solution:

We will use the following results of the Bohr's theory of the hydrogen atom:

Angular momentum, $l = mr^2\omega = n\mathrm{h},\ n = 1, 2, 3..$

$$r = n^2 \frac{\mathrm{h}^2}{m\left(e^2/4\pi\varepsilon_0\right)},\ n = 1, 2, 3..$$

$$E = -\frac{mc^2}{2n^2}\alpha^2,\ n = 1, 2, 3..$$

where

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \mathrm{h}c}.$$

From these results we note that the frequency of rotation of the orbit with principal quantum number n is

$$\nu_n = \frac{mc^2\alpha^2}{n^3 h}.$$

We will consider the limit of large quantum numbers that is we will assume that $n\,?\;1$. The classical frequency of a Bohr orbit with $n\,?\;1$ will therefore be

$$\nu_n = \frac{mc^2\alpha^2}{n^3 h}.$$

We next consider Bohr transitions from the state with principal quantum number n, $n\,?\;1$, to states with principal quantum number $n-n'$, $n'=1,2,3....$

The frequency of spectral lines when hydrogen atom makes transitions from state n, $n\,?\;1$, to states with principal quantum number $n-n'$, $n'=1,2,3....$

Will be

$$\nu_{n'} = \frac{E_n - E_{n-n'}}{h} = \frac{mc^2\alpha^2}{2}\left(\frac{1}{\left(n-n'\right)^2} - \frac{1}{n^2}\right)$$

$$= \frac{mc^2\alpha^2}{2} \times \frac{2nn' - n'^2}{n^2\left(n-n'\right)^2}.$$

We obtain the expression for $\nu_{n'}$ in the limit $n\,?\;1$.

We find

$$\lim_{n?\ 1} \nu_{n'} = \frac{mc^2\alpha^2}{n^3h} \times n',\ n' = 1,2,3...$$

$$= n'\nu_n,\ n' = 1,2,3...$$

We thus note that in the limit of large quantum numbers, the correspondence limit, the quantum result agrees with the classical physics result.