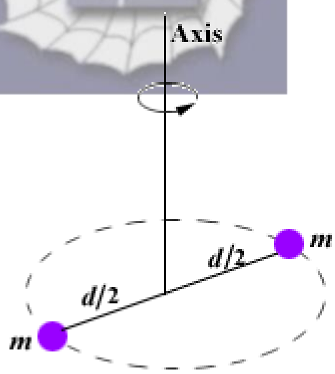
747.

Problem 51.12 (RHK)

A diatomic gas molecule consists of two atoms of mass m *separated by a fixed distance* d *rotating about an axis as indicated in the figure. Assuming that its angular momentum is quantized as in the Bohr atom, we have to determine* (a) *the possible angular velocities and* (b) *the possible rotational energies.* (c) *We have to calculate, according to this model, in* eV, *of an* O_2 *molecule in which* $d = 121\ \text{pm}$ *and* $m = 16.0\ \text{u}$.



Solution:

(a)

Let ω be the angular velocity with which the two atoms, each of mass m, are rotating, as shown in the figure.

The total angular momentum of this system about the axis of rotation will be

$$L = 2\times\left(\frac{md^2\omega}{4}\right) = \frac{1}{2}md^2\omega.$$

Applying the Bohr quantization condition on the angular momentum, we get

$$L = \frac{1}{2}md^2\omega = n\mathrm{h},\; n = 1,2,3,..$$

We thus note that the angular velocities are quantized and are given by the equation

$$\omega = \frac{2n\mathrm{h}}{md^2},\; n = 1,2,3,..$$

(b)

The rotational energies of this system will be determined by the quantized angular velocities as follows:

$$E = 2\times\left(\frac{m}{2}\left(\frac{\omega d}{2}\right)^2\right) = \frac{md^2\omega^2}{4} = \frac{md^2}{4}\times\left(\frac{4n^2\mathrm{h}^2}{m^2d^4}\right)$$

or

$$E_n = \frac{n^2\mathrm{h}^2}{md^2},\; n = 1,2,3...$$

(c)

We will calculate next using this model the ground state energy in eV of the O_2 molecule with $d = 121\text{ pm}$ and $m = 16.0\text{ u}$.

We will use the following data and values:

$$m = 16.0\text{ u} = 16.0 \times 1.66 \times 10^{-27}\text{ kg}$$
$$= 26.56 \times 10^{-27}\text{ kg.}$$

$$d = 121 \times 10^{-12}\text{ m,}$$

and

$$\text{h} = 1.054 \times 10^{-34}\text{ J s}$$

The ground state of the O_2 molecule will correspond to $n = 1$. Substituting the above data in the formula for the quantized rotator, we get

$$E_1 = \frac{\left(1.054 \times 10^{-34}\right)^2}{26.56 \times 10^{-27} \times \left(121 \times 10^{-12}\right)^2}\text{ J}$$

$$= 2.86 \times 10^{-23}\text{ J.}$$

As $1\text{J} = 6.242 \times 10^{18}\text{ eV}$,

we get

$$E_1 = 2.86 \times 10^{-23} \times 6.242 \times 10^{18}\text{ eV}$$
$$= 1.78 \times 10^{-4}\text{ eV.}$$